Series & Sequence

⑥ Find "a", "r", and S_5

$t_n = ar^{n-1}$

$\frac{1}{9} \div 9 = \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$

$t_3 = 9$

$t_3 = ar^{3-1}$

$t_3 = ar^2$

$9 = ar^2$

$ar^2 = 9$

$t_7 = \frac{1}{9}$

$t_7 = ar^{7-1}$

$t_7 = ar^6$

$\frac{1}{9} = ar^6$

$ar^6 = \frac{1}{9}$

Elimination

$$\frac{ar^6 = \frac{1}{9}}{ar^2 = 9}$$

$r^4 = \frac{1}{81}$

$\boxed{r = \pm\frac{1}{3}}$

$ar^2 = 9$

$a\left(\pm\frac{1}{3}\right)^2 = 9$

$a\left(\frac{1}{9}\right) = 9$

$\cancel{9} \cdot \frac{a}{\cancel{9}} = 9 \cdot 9$

$\boxed{a = 81}$

when $r = \frac{1}{3}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_5 = \frac{81\left(\left(\frac{1}{3}\right)^5 - 1\right)}{\frac{1}{3} - 1}$$

$$S_5 = \frac{81\left(\frac{1}{243} - \frac{243}{243}\right)}{\frac{1}{3} - \frac{3}{3}}$$

$$S_5 = 81\left(\frac{-242}{243}\right) \div \frac{-2}{3}$$

$$S_5 = \cancel{81}^{121}\left(\frac{-\cancel{242}}{\cancel{243}}\right)\left(\frac{-\cancel{3}}{\cancel{2}}\right)$$

$S_5 = 121$

when $r = -\frac{1}{3}$

$$S_5 = \frac{81\left(\left(-\frac{1}{3}\right)^5 - 1\right)}{-\frac{1}{3} - 1}$$

$$S_5 = \frac{81\left(\frac{-1}{243} - \frac{243}{243}\right)}{-\frac{1}{3} - \frac{3}{3}}$$

$$S_5 = \cancel{81}^{61}\left(\frac{-\cancel{244}}{\cancel{243}}\right)\left(-\frac{\cancel{3}}{\cancel{4}}\right)$$

$S_5 = 61$

Series & Sequence

$t_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

① $t_{12} = 15$

$t_{12} = a + (12-1)d$

$t_{12} = a + 11d$

$15 = a + 11d$

$\boxed{a + 11d = 15}$

$S_{15} = 105$

$S_{15} = \frac{15}{2}[2a + (15-1)d]$

$S_{15} = 7.5[2a + 14d]$

$S_{15} = 15a + 105d$

$105 = 15a + 105d$

$15a + 105d = 105$

$\boxed{a + 7d = 7}$

Elimination

$a + 11d = 15$

$(-)\ a + 7d = 7$

$\frac{4d}{4} = \frac{8}{4}$

$\boxed{d = 2}$

↳

$a + 7d = 7$

$a + 7(2) = 7$

$a + 14 = 7$

$\boxed{a = -7}$

$t_1 = -7$

$t_2 = -5$

$t_3 = -3$

binomial theore

- used to expand $(x+y)^n$, $n \in \mathrm{N}$
- each term has the f $_nC_k(x)^{n-k}(y)^k$, where $k+1$ is the term number

You can use the **binomial theorem** to expand any power of a binomial expression.

$$(x+y)^n = {}_nC_0(x)^n(y)^0 + {}_nC_1(x)^{n-1}(y)^1 + {}_nC_2(x)^{n-2}(y)^2 + \cdots + {}_nC_{n-1}(x)^1(y)^{n-1} + {}_nC_n(x)^0(y)^n$$

In this chapter, all binomial expansions will be written in descending order of the exponent of the first term in the binomial.

The following are some important observations about the expansion of $(x + y)^n$, where x and y represent the terms of the binomial and $n \in \mathrm{N}$:

- the expansion contains $n + 1$ terms
- the number of objects, k, selected in the combination $_nC_k$ can be taken to match the number of factors of the second variable selected; that is, it is the same as the exponent on the second variable
- the general term, t_{k+1}, has the form

$$_nC_k(x)^{n-k}(y)^k$$

the same

- the sum of the exponents in any term of the expansion is n

Permutations + Combinations

① 3 digit # from 0, 1, 2, 8, and 9
with no repititions

$\underline{4} \times \underline{4} \times \underline{3} = 48$ C

can't
use 0
as digit
one

⑮ $\left(y - \frac{2}{y^2}\right)^5$

$n = 5$
$x = y$
$y = \frac{-2}{y^2}$

$_5C_0(y)^5\left(\frac{-2}{y^2}\right)^0 + {_5C_1}(y)^4\left(\frac{-2}{y^2}\right)^1 + {_5C_2}(y)^3\left(\frac{-2}{y^2}\right)^2 + {_5C_3}(y)^2\left(\frac{-2}{y^2}\right)^3 + {_5C_4}(y)^1\left(\frac{-2}{y^2}\right)^4 + {_5C_5}(y)^0\left(\frac{-2}{y^2}\right)^5$

$1(y^5)(1) + 5(y^4)\left(\frac{-2}{y^2}\right) + 10(y^3)\left(\frac{4}{y^4}\right) + 10(y^2)\left(\frac{-8}{y^6}\right) + 5(y)\left(\frac{16}{y^8}\right) + 1(1)\left(\frac{-32}{y^{10}}\right)$

$y^5 - \frac{10y^4}{y^2} + \frac{40y^3}{y^4} - \frac{80y^2}{y^8} + \frac{80y}{y^8} - \frac{32}{y^{10}}$

$$\boxed{y^5 - 10y^2 + \frac{40}{y} - \frac{80}{y^4} + \frac{80}{y^7} - \frac{32}{y^{10}}}$$

5. $(2x^2+3y)^7 \rightarrow 3^{rd}$ term

$${}_7C_2(2x^2)^5(3y)^2$$

$$21(32x^{10})(9y^2)$$

$$\boxed{6048x^{10}y^2}$$

③ $$y = \sqrt[7]{2x^2 + \sqrt{x^7 - 8x\sqrt{3-x}}} = \left[2x^2 + \left(x^7 - 8x(3-x)^{1/2}\right)^{1/2}\right]^{1/7}$$

$$y' = \frac{1}{7}\left[2x^2 + \left(x^7 - 8x(3-x)^{1/2}\right)^{1/2}\right]^{-6/7}\left[4x + \frac{1}{2}\left(x^7 - 8x(3-x)^{1/2}\right)^{-1/2}\left(7x^6 - \left(8(3-x)^{1/2} + 8x\left(\frac{1}{2}\right)(3-x)^{-1/2}(-1)\right)\right)\right]$$

(2) Let $f(x) = \begin{cases} -x^2+2, & \text{if } x<1 \\ 1, & \text{if } x=1 \\ -2x+3, & \text{if } 1<x\leq 3 \\ -3, & \text{if } x>3 \end{cases}$

$-x^2+2$

x	y
∘ 1	1
0	2
-1	1
-2	-2

1

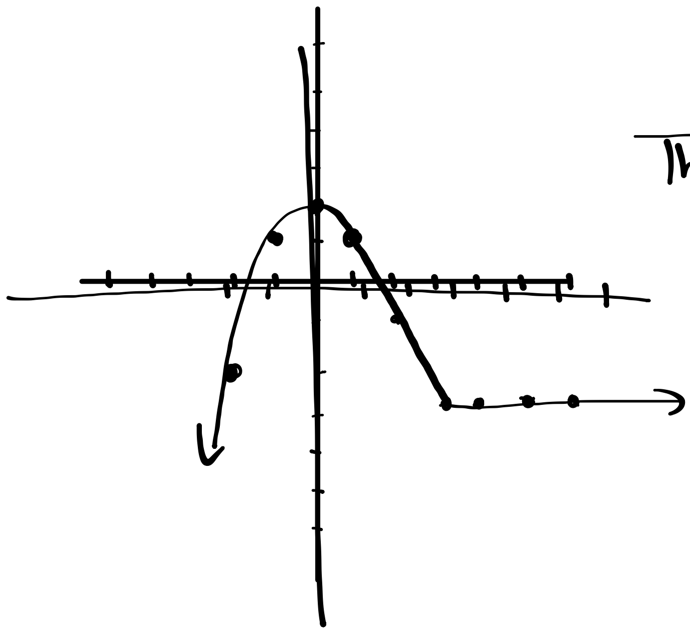
x	y
• 1	1

$-2x+3$

x	y
∘ 1	1
2	-1
• 3	-3

-3

x	y
∘ 3	-3
4	-3
5	-3
6	-3



The function is continuous

② $\left(x^2 - \frac{x}{2}\right)^5$

$${}_5C_0(x^2)^5\left(-\frac{x}{2}\right)^0 + {}_5C_1(x^2)^4\left(-\frac{x}{2}\right)^1 + {}_5C_2(x^2)^3\left(-\frac{x}{2}\right)^2 + {}_5C_3(x^2)^2\left(-\frac{x}{2}\right)^3 + {}_5C_4(x^2)^1\left(-\frac{x}{2}\right)^4 + {}_5C_5(x^2)^0\left(-\frac{x}{2}\right)^5$$

$$1(x^{10})(1) + 5(x^8)\left(\frac{-x}{2}\right) + 10(x^6)\left(\frac{x^2}{4}\right) + 10(x^4)\left(\frac{-x^3}{8}\right) + 5(x^2)\left(\frac{x^4}{16}\right) + 1(1)\left(\frac{-x^5}{32}\right)$$

$$x^{10} - \frac{5}{2}x^9 + \frac{5}{2}x^8 - \frac{5}{4}x^7 + \frac{5}{16}x^6 - \frac{x^5}{32}$$

4 c) Case 1 (All Black)

$${}_{26}C_5 = \boxed{65\,780}$$

Case 2 (4 Black + 1 Red)

$${}_{26}C_4 \times {}_{26}C_1 = 14\,950 \times 26 = \boxed{388\,700}$$

Case 3 (3 black + 2 red)

$${}_{26}C_3 \times {}_{26}C_2 = 2600 \times 325 = \boxed{845000}$$

Total $= 845\,000 + 388\,700 + 65\,780 = 1\,299\,480$

5 14 Letters

- 5 are burnt
- 9 are good

3 good & 2 bad

$_9C_3 \times {}_5C_2$

84×10

840

Derivatives Exam Review:

4 b) $y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{(3+x^2)}$

$$y' = \frac{(3+x^2)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(2x)}{(3+x^2)^2}$$

$$y' = \frac{2\cdot\frac{3}{2}x^{-1/2} + 2\cdot\frac{1}{2}x^{3/2} - 2\cdot 2x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{3x^{-1/2} + x^{3/2} - 4x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{x^{-1/2}(3 + x^2 - 4x^2)}{2(3+x^2)^2}$$

$$y' = \frac{3-3x^2}{2x^{1/2}(3+x^2)^2} = \frac{3(1-x^2)}{2\sqrt{x}(3+x^2)^2}$$

5 c) $1 + 5 + 9 + \ldots + 77$

$\underbrace{1 + 5}_{4} \underbrace{+ 9}_{4}$

$a = 1$

$t_n = 77$

$d = 4$

① Find n:

$t_n = a + (n-1)d$

$77 = 1 + (n-1)4$

$76 = 4(n-1)$

$19 = n-1$

$\boxed{20 = n}$

② Find S_{20}

$S_{20} = \frac{20}{2}[1+77]$

$= 10(78)$

$\boxed{= 780}$

5 b) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1} = 3 + \frac{3}{2} + \frac{3}{4} + \ldots$

$\frac{1}{2}$ $\frac{1}{2}$

$a = 3$

$r = \frac{1}{2}$

$S_n = \frac{3}{1-\frac{1}{2}} = \frac{3}{\frac{1}{2}} = 3 \cdot 2 = 6$

4 c) $\lim_{n \to \infty} (-1)^{n+1} n^2$

$= 1, -4, 9, -16, 25, -36, 49, -64, 81, -100$

= Diverging (has no limit)

$\_\_\_, \frac{1}{5}, \_\_\_, \_\_\_, \frac{25}{\quad}, \_\_\_, \_\_\_$

$t_2 = \frac{1}{5}$, $t_5 = 25$

$t_2 = ar^{2-1}$ $t_5 = ar^{5-1}$

$t_2 = ar^1$ $t_5 = ar^4$

$\frac{1}{5} = ar$ $25 = ar^4$

$$\boxed{ar = \frac{1}{5} \qquad ar^4 = 25}$$

$$\frac{ar^4 = 25}{ar = 1/5}$$

$r^3 = 125$

$r = 5$

$ar = 1/5$

$a(5) = \frac{1}{5}$

$5a = \frac{1}{5}$

$a = \frac{1}{25}$

Expand: $\left(x-\frac{1}{2}\right)^{4}$

$${}_4C_0(x)^4\left(-\tfrac{1}{2}\right)^0+{}_4C_1(x)^3\left(-\tfrac{1}{2}\right)^1+{}_4C_2(x)^2\left(-\tfrac{1}{2}\right)^2+{}_4C_3(x)^1\left(-\tfrac{1}{2}\right)^3+{}_4C_4(x)^0\left(-\tfrac{1}{2}\right)^4$$

$$(1)(x^4)(1)+(4)(x^3)\left(-\frac{1}{2}\right)+(6)(x^2)\left(\frac{1}{4}\right)+(4)(x)\left(-\frac{1}{8}\right)+(1)(1)\left(\frac{1}{16}\right)$$

$$x^4-\frac{4x^3}{2}+\frac{6x^2}{4}-\frac{4x}{8}+\frac{1}{16}$$

$$\boxed{x^4-2x^3+\frac{3}{2}x^2-\frac{1}{2}x+\frac{1}{16}}$$

$f(x) = \boxed{3x^2 + 2x - 7}$

$f(x+h) = 3(x+h)^2 + 2(x+h) - 7$

$= 3(x^2 + 2xh + h^2) + 2x + 2h - 7$

$= \boxed{3x^2 + 6xh + 3h^2 + 2x + 2h - 7}$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{2x} + 2h \cancel{-7} - (\cancel{3x^2} + \cancel{2x} \cancel{-7})}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(6x + 3h + 2)}{\cancel{h}} = \boxed{6x + 2}$$

$$f(x) = \frac{(3x^2+5)^3}{\sqrt{2x-7}}$$

$$\frac{f'g - fg'}{g^2}$$

$$f'(x) = \frac{\overbrace{3(3x^2+5)^2(6x)}^{f'}\overbrace{(2x-7)^{1/2}}^{g} - \overbrace{(3x^2+5)^3}^{f}\overbrace{\left(\tfrac{1}{2}\right)(2x-7)^{-1/2}(2)}^{g'}}{\left[\sqrt{2x-7}\right]^2}$$

$$f'(x) = \frac{18x(3x^2+5)^2(2x-7)^{1/2} - (3x^2+5)^3(2x-7)^{-1/2}}{2x-7}$$ ← Factor

$$36x^2 - 126x - 3x^2 - 5$$

$$f'(x) = \frac{(3x^2+5)^2(2x-7)^{-1/2}\left[18x(2x-7) - (3x^2+5)\right]}{(2x-7)}$$

$$f'(x) = \frac{(3x^2+5)^2(33x^2-126x-5)}{(2x-7)^{3/2}}$$

$$\lim_{x\to 0} \frac{\cancel{3(x+3)}\frac{1}{\cancel{x+3}} - \frac{1}{\cancel{3}}\cancel{3}(x+3)}{x \cdot 3(x+3)}$$

CD: $3(x+3)$

$$\lim_{x\to 0} \frac{3-(x+3)}{3x(x+3)}$$

$$\lim_{x\to 0} \frac{3-x-3}{3x(x+3)}$$

$$\lim_{x\to 0} \frac{-\cancel{x}}{3\cancel{x}(x+3)} = \frac{-1}{3(3)} = \frac{-1}{9}$$