Ch. 1 Practice Test

(10) 5, 36, 67, 98, 129, 160 (Arithmetic)

Given:

$t_1 = 5$, $t_6 = 160$

$t_n = a + (n-1)d$

$t_1 = a + (1-1)d$

$t_1 = a + 0d$

$t_1 = a$

$\boxed{5 = a}$

$t_n = a + (n-1)d$

$t_6 = a + (6-1)d$

$t_6 = a + 5d$

$\boxed{160 = a + 5d}$

$160 = a + 5d$

$160 = 5 + 5d$

$\frac{155}{5} = \frac{5d}{5}$

$\boxed{31 = d}$

b) general term

$t_n = a + (n-1)d$

$t_n = 5 + (n-1)(31)$

$t_n = 5 + 31n - 31$

$\boxed{t_n = 31n - 26}$

Series + Sequence

① $\underline{80000}$, ___, ___, ___, $\underline{117128}$

$a = 80000$

$t_5 = 117128$

$n = 5$

$r = ?$

$ARoI = ?$

$t_n = ar^{n-1}$

$\frac{117128}{80000} = \frac{\cancel{80000}r^{5-1}}{\cancel{80000}}$

$(1.4641)^{\frac{1}{4}} = (r^4)^{\frac{1}{4}}$

$\boxed{1.1 = r}$

$\sqrt[4]{1.4641} = \sqrt[4]{r^4}$

$\pm 1.1 = r$

$1.1 = r$

$ARoI = 100(1.1 - 1) = 10\%$

Series + Sequence

5) a) $\sum_{n=1}^{5} n^2+1$

$= [(1)^2+1] + [(2)^2+1] + [(3)^2+1] + [(4)^2+1] + [(5)^2+1]$

$= 2+5+10+17+26$

$= 60$

b) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$

$a = 3$

$r = 1/2$

$S_n = \frac{a}{1-r}$

$S_n = \frac{3}{1-1/2}$

$S_n = \frac{3}{2/2-1/2}$

$S_n = \frac{3}{1/2} = 3 \times 2 = 6$

Series & Sequence

5 c) $1+5+9+\ldots+77$

d: 4, 4

$a=1$

$t_n=77$

$d=4$

① Find n:

$t_n=a+(n-1)d$

$77=1+(n-1)4$

$76=4(n-1)$

$19=n-1$

$\boxed{20=n}$

② Find S_{20}

$S_{20}=\frac{20}{2}[1+77]$

$=10(78)$

$\boxed{=780}$

5 b) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1} = 3+\frac{3}{2}+\frac{3}{4}+\ldots$

r: ½, ½

$a=3$

$r=\frac{1}{2}$

$S_n=\frac{3}{1-\frac{1}{2}}=\frac{3}{\frac{1}{2}}=3\cdot 2=6$

4 c) $\lim_{n\to\infty} (-1)^{n+1} n^2$

$=1, -4, 9, -16, 25, -36, 49, -64, 81, -100$

= Diverging (has no limit)

Series + Sequence

⑥ Find "a", "r", and S_5

$t_n = ar^{n-1}$

$\frac{1}{9} \div 9 = \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$

$t_3 = 9$

$t_3 = ar^{3-1}$

$t_3 = ar^2$

$9 = ar^2$

$ar^2 = 9$

$t_7 = \frac{1}{9}$

$t_7 = ar^{7-1}$

$t_7 = ar^6$

$\frac{1}{9} = ar^6$

$ar^6 = \frac{1}{9}$

Elimination

$\frac{ar^6 = \frac{1}{9}}{ar^2 = 9}$

$r^4 = \frac{1}{81}$

$\boxed{r = \pm\frac{1}{3}}$

$ar^2 = 9$

$a\left(\pm\frac{1}{3}\right)^2 = 9$

$a\left(\frac{1}{9}\right) = 9$

$\cancel{9} \cdot \frac{a}{\cancel{9}} = 9 \cdot 9$

$\boxed{a = 81}$

when $r = \frac{1}{3}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_5 = \frac{81\left(\left(\frac{1}{3}\right)^5 - 1\right)}{\frac{1}{3} - 1}$

$S_5 = \frac{81\left(\frac{1}{243} - \frac{243}{243}\right)}{\frac{1}{3} - \frac{3}{3}}$

$S_5 = 81\left(\frac{-242}{243}\right) \div -\frac{2}{3}$

$S_5 = \cancel{81}^{121}\left(\frac{-\cancel{242}}{\cancel{243}}\right)\left(\frac{-\cancel{3}}{\cancel{2}}\right)$

$S_5 = 121$

when $r = -\frac{1}{3}$

$S_5 = \frac{81\left(\left(-\frac{1}{3}\right)^5 - 1\right)}{-\frac{1}{3} - 1}$

$S_5 = \frac{81\left(\frac{-1}{243} - \frac{243}{243}\right)}{-\frac{1}{3} - \frac{3}{3}}$

$S_5 = \cancel{81}^{61}\left(\frac{-\cancel{244}}{\cancel{243}}\right)\left(-\frac{\cancel{3}}{\cancel{4}}\right)$

$S_5 = 61$

Series & Sequence

$t_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

① $t_{12} = 15$

$t_{12} = a + (12-1)d$

$t_{12} = a + 11d$

$15 = a + 11d$

$\boxed{a + 11d = 15}$

$S_{15} = 105$

$S_{15} = \frac{15}{2}[2a + (15-1)d]$

$S_{15} = 7.5[2a + 14d]$

$S_{15} = 15a + 105d$

$105 = 15a + 105d$

$15a + 105d = 105$

$\boxed{a + 7d = 7}$

Elimination

$a + 11d = 15$

$(-)\ a + 7d = 7$

$\frac{4d}{4} = \frac{8}{4}$

$\boxed{d = 2}$

↳

$a + 7d = 7$

$a + 7(2) = 7$

$a + 14 = 7$

$\boxed{a = -7}$

$t_1 = -7$

$t_2 = -5$

$t_3 = -3$

Series & Sequence

$\_\_\_,\ \frac{1}{5},\ \_\_\_,\ \_\_,\ 25,\ \_\_\_,\ \_\_\_$

$t_2 = \frac{1}{5}$, $t_5 = 25$

$t_2 = ar^{2-1}$ $\quad$ $t_5 = ar^{5-1}$

$t_2 = ar^1$ $\quad$ $t_5 = ar^4$

$\frac{1}{5} = ar$ $\quad$ $25 = ar^4$

$$\boxed{ar = \frac{1}{5} \qquad ar^4 = 25}$$

$$\frac{ar^4 = 25}{ar = 1/5}$$

$r^3 = 125$

$r = 5$

$ar = 1/5$

$a(5) = \frac{1}{5}$

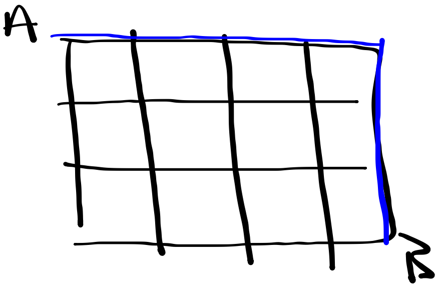
$5a = \frac{1}{5}$

$a = \frac{1}{25}$

Permutations + Combinations

MISSISSIPPI $n=11$

$$\frac{11!}{4!\,4!\,2!} =$$



RRRRDDD $n=7$

$$\frac{7!}{4!\,3!} =$$

Permutations + Combinations

binomial theore

- used to expand $(x + y)^n$, $n \in \mathrm{N}$
- each term has the f $_nC_k(x)^{n-k}(y)^k$, where $k + 1$ is the term number

You can use the **binomial theorem** to expand any power of a binomial expression.

$$(x + y)^n = {}_nC_0(x)^n(y)^0 + {}_nC_1(x)^{n-1}(y)^1 + {}_nC_2(x)^{n-2}(y)^2 + \cdots + {}_nC_{n-1}(x)^1(y)^{n-1} + {}_nC_n(x)^0(y)^n$$

In this chapter, all binomial expansions will be written in descending order of the exponent of the first term in the binomial.

The following are some important observations about the expansion of $(x + y)^n$, where x and y represent the terms of the binomial and $n \in \mathrm{N}$:

- the expansion contains $n + 1$ terms
- the number of objects, k, selected in the combination $_nC_k$ can be taken to match the number of factors of the second variable selected; that is, it is the same as the exponent on the second variable
- the general term, t_{k+1}, has the form

$$_nC_k(x)^{n-k}(y)^k$$

the same

- the sum of the exponents in any term of the expansion is n

Permutations + Combinations

① 3 digit # from 0, 1, 2, 8, and 9 with no repititions

$\underline{4} \times \underline{4} \times \underline{3} = 48$ C

can't use 0 as digit one

⑮ $\left(y - \frac{2}{y^2}\right)^5$

$n = 5$
$x = y$
$y = \frac{-2}{y^2}$

$${}_5C_0(y)^5\left(\frac{-2}{y^2}\right)^0 + {}_5C_1(y)^4\left(\frac{-2}{y^2}\right)^1 + {}_5C_2(y)^3\left(\frac{-2}{y^2}\right)^2 + {}_5C_3(y)^2\left(\frac{-2}{y^2}\right)^3 + {}_5C_4(y)^1\left(\frac{-2}{y^2}\right)^4 + {}_5C_5(y)^0\left(\frac{-2}{y^2}\right)^5$$

$$1(y^5)(1) + 5(y^4)\left(\frac{-2}{y^2}\right) + 10(y^3)\left(\frac{4}{y^4}\right) + 10(y^2)\left(\frac{-8}{y^6}\right) + 5(y)\left(\frac{16}{y^8}\right) + 1(1)\left(\frac{-32}{y^{10}}\right)$$

$$y^5 - \frac{10y^4}{y^2} + \frac{40y^3}{y^4} - \frac{80y^2}{y^6} + \frac{80y}{y^8} - \frac{32}{y^{10}}$$

$$\boxed{y^5 - 10y^2 + \frac{40}{y} - \frac{80}{y^4} + \frac{80}{y^7} - \frac{32}{y^{10}}}$$

Permutations + Combinations

5 $(2x^2+3y)^7 \rightarrow 3^{rd}$ term

$_7C_2(2x^2)^5(3y)^2$

$21(32x^{10})(9y^2)$

$\boxed{6048x^{10}y^2}$

Permutations + Combinations

② $\left(x^2 - \frac{x}{2}\right)^5$

$${}_5C_0(x^2)^5\left(-\frac{x}{2}\right)^0 + {}_5C_1(x^2)^4\left(-\frac{x}{2}\right)^1 + {}_5C_2(x^2)^3\left(-\frac{x}{2}\right)^2 + {}_5C_3(x^2)^2\left(-\frac{x}{2}\right)^3 + {}_5C_4(x^2)^1\left(-\frac{x}{2}\right)^4 + {}_5C_5(x^2)^0\left(-\frac{x}{2}\right)^5$$

$$1(x^{10})(1) + 5(x^8)\left(\frac{-x}{2}\right) + 10(x^6)\left(\frac{x^2}{4}\right) + 10(x^4)\left(\frac{-x^3}{8}\right) + 5(x^2)\left(\frac{x^4}{16}\right) + 1(1)\left(\frac{-x^5}{32}\right)$$

$$x^{10} - \frac{5x^9}{2} + \frac{5}{2}x^8 - \frac{5}{4}x^7 + \frac{5}{16}x^6 - \frac{x^5}{32}$$

Perm/Comb

(2) $\left(x^2 - \frac{x}{3}\right)^3$ $a = x^2$ $b = -\frac{x}{3}$ $n = 3$

$_3C_0(x^2)^3\left(-\frac{x}{3}\right)^0 + {}_3C_1(x^2)^2\left(-\frac{x}{3}\right)^1 + {}_3C_2(x^2)^1\left(-\frac{x}{3}\right)^2 + {}_3C_3(x^2)^0\left(-\frac{x}{3}\right)^3$

$1(x^6)(1) + 3(x^4)\left(-\frac{x}{3}\right) + 3(x^2)\left(\frac{x^2}{9}\right) + 1(1)\left(\frac{-x^3}{27}\right)$

$x^6 - \frac{3x^5}{3} + \frac{3x^4}{9} - \frac{1x^3}{27}$

$\boxed{x^6 - x^5 + \frac{1}{3}x^4 - \frac{1}{27}x^3}$

Permutations + Combinations

Expand: $\left(x-\frac{1}{2}\right)^{4}$

$${}_4C_0(x)^4\left(-\frac{1}{2}\right)^0+{}_4C_1(x)^3\left(-\frac{1}{2}\right)^1+{}_4C_2(x)^2\left(-\frac{1}{2}\right)^2+{}_4C_3(x)^1\left(-\frac{1}{2}\right)^3+{}_4C_4(x)^0\left(-\frac{1}{2}\right)^4$$

$$(1)(x^4)(1)+(4)(x^3)\left(-\frac{1}{2}\right)+(6)(x^2)\left(\frac{1}{4}\right)+(4)(x)\left(-\frac{1}{8}\right)+(1)(1)\left(\frac{1}{16}\right)$$

$$x^4-\frac{4x^3}{2}+\frac{6x^2}{4}-\frac{4x}{8}+\frac{1}{16}$$

$$\boxed{x^4-2x^3+\frac{3}{2}x^2-\frac{1}{2}x+\frac{1}{16}}$$

Permutations + Combinations

4 c) Case 1 (All Black)

$_{26}C_5 = \boxed{65\ 780}$

Case 2 (4 Black + 1 Red)

$_{26}C_4 \times {}_{26}C_1 = 14\ 950 \times 26 = \boxed{388\ 700}$

Case 3 (3 black + 2 red)

$_{26}C_3 \times {}_{26}C_2 = 2600 \times 325 = \boxed{845000}$

Total $= 845\ 000 + 388\ 700 + 65\ 780 = 1\ 299\ 480$

Permutations + Combinations

5 14 Letters
- 5 are burnt
- 9 are good

3 good + 2 bad

$_9C_3 \times {_5C_2}$

84×10

840

Limit Review:

$x^2 - 25$

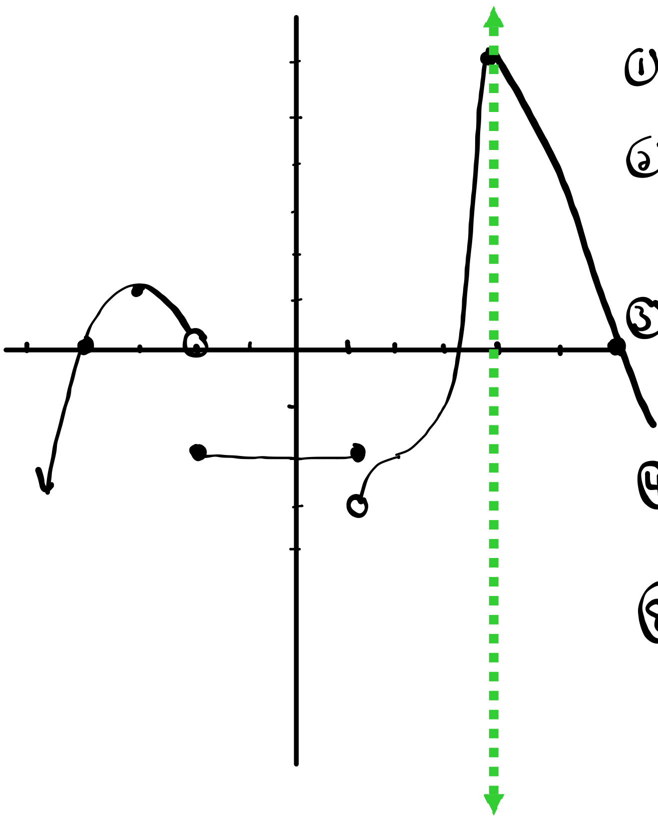
$(x+5)(x-5)$

① g) $\lim_{x \to b} \frac{x^2 - b^2}{x^8 - b^8}$

$\lim_{x \to b} \frac{(x^2 - b^2)}{(x^4 + b^4)(x^4 - b^4)}$

$\lim_{x \to b} \frac{\cancel{(x^2 - b^2)}}{(x^4 + b^4)(x^2 + b^2)\cancel{(x^2 - b^2)}} = \frac{1}{(2b^4)(2b^2)} = \frac{1}{4b^6}$

d) $\lim_{x \to 0} \frac{x^3 + 1}{x^2 + 3x + 2} = \frac{1}{2}$



① $f(-2) = -2$

② $\lim_{x \to -2^-} f(x) = 0$

③ $\lim_{x \to 1^+} f(x) = -3$

④ $f(4) = 6$

⑧ $f(1) = -2$

Limits

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

Limits

① c) $\lim_{a \to b} \frac{(a+2b)^2 - 9b^2}{a-b}$

$\lim_{a \to b} \frac{(a+2b+3b)(a+2b-3b)}{(a-b)}$

$\lim_{a \to b} \frac{(a+5b)\cancel{(a-b)}}{\cancel{(a-b)}} = 6b$

① f) $\lim_{x \to 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{(x^2+5x)(5(x+5))}$ CD: $5(x+5)$

$\lim_{x \to 0} \frac{5 - 1(x+5)}{x(x+5)(5(x+5))}$

$\lim_{x \to 0} \frac{5 - x - 5}{5x(x+5)^2}$

$\lim_{x \to 0} \frac{-\cancel{x}}{5\cancel{x}(x+5)^2} = \frac{-1}{5(25)} = \frac{-1}{125}$

Limits

2. Let $f(x) = \begin{cases} -x^2+2, & \text{if } x<1 \\ 1, & \text{if } x=1 \\ -2x+3, & \text{if } 1<x\leq 3 \\ -3, & \text{if } x>3 \end{cases}$

$-x^2+2$

x	y
1	1
0	2
-1	1
-2	-2

1

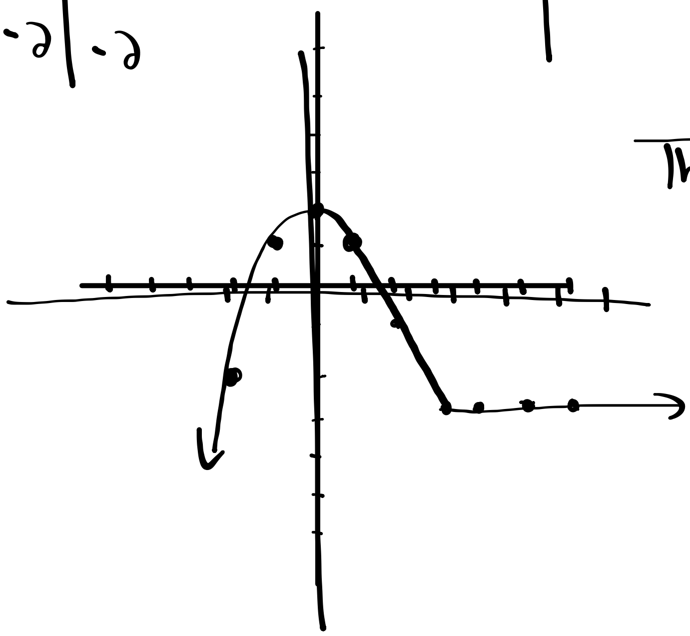
x	y
1	1

$-2x+3$

x	y
1	1
2	-1
3	-3

-3

x	y
3	-3
4	-3
5	-3
6	-3



The function is continuous

Limits

$$\lim_{x\to 0} \frac{\cancel{3(x+3)}\,\frac{1}{\cancel{x+3}} - \frac{1}{\cancel{3}}\,\cancel{3}(x+3)}{x\;3(x+3)}$$

CD: $3(x+3)$

$$\lim_{x\to 0} \frac{3-(x+3)}{3x(x+3)}$$

$$\lim_{x\to 0} \frac{3-x-3}{3x(x+3)}$$

$$\lim_{x\to 0} \frac{-\cancel{x}}{3\cancel{x}(x+3)} = \frac{-1}{3(3)} = -\frac{1}{9}$$

Differentiation

⑤ $y=(x^2-3)^8$ @ $x=2$

(i) Find y:

$y=(2^2-3)^8$

$y=(4-3)^8$

$y=\underline{1}$

(ii) Find y':

$y=(x^2-3)^8$

$y'=8(x^2-3)^7(2x)$

$y'=16x(x^2-3)^7$

(iii) Find slope (m):

$y'=16(2)[(2)^2-3]^7$

$y'=32[4-3]^7$

$y'=32$ ← $m=32$

(iv): $y-y_1=m(x-x_1)$

$y-1=32(x-2)$

$y-1=32x-64$

$\boxed{y=32x-63}$

or $\boxed{32x-y-63=0}$

⑦ a) $f(x)=\left(\dfrac{2x+1}{x-1}\right)^5$

$$f'(x)=5\left(\frac{2x+1}{x-1}\right)^4\left[\frac{2(x-1)-1(2x+1)}{(x-1)^2}\right]$$

$2x-2 \quad -2x-1$

$$f'(x)=5\frac{(2x+1)^4}{(x-1)^4}\cdot\frac{-3}{(x-1)^2}$$

$$f'(x)=\frac{-15(2x+1)^4}{(x-1)^6}$$

Differentiation

Product: $(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$

$$y = \underbrace{(3x^2-5)}_{f(x)}\underbrace{(4x^3+3x)}_{g(x)}$$

$$y' = 6x(4x^3+3x) + (3x^2-5)(12x^2+3)$$

$$y' = 24x^4 + 18x^2 + 36x^4 + 9x^2 - 60x^2 - 15$$

$$\boxed{y' = 60x^4 - 33x^2 - 15}$$

Quotient: $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$$y = \frac{x^2+2 \leftarrow f(x)}{3x+5 \leftarrow g(x)}$$

$$y' = \frac{2x(3x+5) - 3(x^2+2)}{(3x+5)^2}$$

$$y' = \frac{6x^2+10x-3x^2-6}{(3x+5)^2} = \boxed{\frac{3x^2+10x-6}{(3x+5)^2}}$$

Chain: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

$$y = \sqrt{4x^3-6x} = (4x^3-6x)^{1/2}$$

$$y' = \frac{1}{2}(4x^3-6x)^{-1/2}(12x^2-6)$$

$$y' = \frac{12x^2-6}{2(4x^3-6x)^{1/2}} = \frac{\cancel{6}^{\,3}(2x^2-1)}{\cancel{2}\sqrt{4x^3-6x}} = \boxed{\frac{3(2x^2-1)}{\sqrt{4x^3-6x}}}$$

$$\frac{6x^2-3}{\sqrt{4x^3-6x}}$$

Combo:

$$y = (3x^2+5)^3\sqrt{4x-2} = \underbrace{(3x^2+5)^3}_{f(x)}\underbrace{(4x-2)^{1/2}}_{g(x)}$$

$$y' = \underbrace{3(3x^2+5)^2(6x)}_{f'(x)}\underbrace{(4x-2)^{1/2}}_{g(x)} + \underbrace{(3x^2+5)^3}_{f(x)}\underbrace{\left(\frac{1}{2}\right)(4x-2)^{-1/2}(4)}_{g'(x)}$$

$$y' = 18x(3x^2+5)^2(4x-2)^{1/2} + 2(3x^2+5)^3(4x-2)^{-1/2}$$

$$\cdot\ 36x^2 - 18x + 3x^2 + 5$$

$$y' = 2(3x^2+5)^2(4x-2)^{-1/2}\left[9x(4x-2) + (3x^2+5)\right]$$

$$y' = 2(3x^2+5)^2(4x-2)^{-1/2}(39x^2-18x+5)$$

$$y' = \frac{2(3x^2+5)^2(39x^2-18x+5)}{\sqrt{4x-2}}$$

$$\frac{(3x^2+5)^2}{(3x^2+5)^2} = (3x^2+5)^0 = 1 \quad \Bigg| \quad \frac{(4x-2)^{1/2}}{(4x-2)^{-1/2}} = (4x-2)^{1/2-(-1/2)} = (4x-2)^1$$

Differentiation

$-\frac{1}{2}\times 3 = \frac{-3}{2}$

$-\frac{1}{2}-1$

$-\frac{1}{2}-\frac{2}{2}=\frac{-3}{2}$

② b) $f(x)=\frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$

$f'(x) = -\frac{3}{2}x^{-3/2} = \frac{-3}{2x^{3/2}}$

④ b) $y=\frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{3+x^2}\begin{matrix} f \\ g\end{matrix}$

$\frac{f'g-fg'}{(g)^2}$

$y'=\frac{\frac{1}{2}x^{-1/2}(3+x^2)-x^{1/2}(2x)}{(3+x^2)^2}$

$x^{1/2}\cdot x^1$

$x^{1/2+1}$

$x^{1/2+2/2}$

$x^{3/2}$

$y'=\frac{\frac{1}{2x^{1/2}}(3+x^2)-2x^{3/2}}{(3+x^2)^2}$

$y'=\frac{\cancel{2x^{1/2}}\frac{(3+x^2)}{\cancel{2x^{1/2}}}-2x^{3/2}\cdot 2x^{1/2}}{2x^{1/2}(3+x^2)^2}$

Complex Fraction:

CD: $2x^{1/2}$

$2x^{3/2}\cdot 2x^{1/2}$

$4x^{3/2+1/2}=4x^{4/2}$

$y'=\frac{3+x^2-4x^2}{2\sqrt{x}(3+x^2)^2}$

$y'=\frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}$

⑥ b) $y=\frac{16}{\sqrt{x-1}} = \frac{16}{(x-1)^{1/2}} = 16(x-1)^{-1/2}$

$y'=-8(x-1)^{-3/2}(1) = -8(x-1)^{-3/2} = \frac{-8}{(x-1)^{3/2}}$

$=\frac{-8}{\sqrt{(x-1)^3}}$

Differentiation

$f(x) = \boxed{3x^2 + 2x - 7}$

$$f(x+h) = 3(x+h)^2 + 2(x+h) - 7$$
$$= 3(x^2 + 2xh + h^2) + 2x + 2h - 7$$
$$= \boxed{3x^2 + 6xh + 3h^2 + 2x + 2h - 7}$$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{+ 2x} + 2h \cancel{- 7} - (\cancel{3x^2} \cancel{+ 2x} \cancel{- 7})}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(6x + 3h + 2)}{\cancel{h}} = \boxed{6x + 2}$$

Differentiation

$$f(x) = \frac{(3x^2+5)^3}{\sqrt{2x-7}} \qquad \frac{f'g - fg'}{g^2}$$

$$f'(x) = \frac{\overbrace{3(3x^2+5)^2(6x)}^{f'}\overbrace{(2x-7)^{1/2}}^{g} - \overbrace{(3x^2+5)^3}^{f}\overbrace{\left(\tfrac{1}{2}\right)(2x-7)^{-1/2}(2)}^{g'}}{\left[\sqrt{2x-7}\right]^2}$$

$$f'(x) = \frac{18x(3x^2+5)^2(2x-7)^{1/2} - (3x^2+5)^3(2x-7)^{-1/2}}{2x-7} \quad \leftarrow \text{Factor}$$

$$36x^2 - 126x - 3x^2 - 5$$

$$f'(x) = \frac{(3x^2+5)^2(2x-7)^{-1/2}\left[18x(2x-7) - (3x^2+5)\right]}{(2x-7)}$$

$$f'(x) = \frac{(3x^2+5)^2(33x^2-126x-5)}{(2x-7)^{3/2}}$$

Differentiation

③ $y = \sqrt[7]{2x^2 + \sqrt{x^7 - 8x\sqrt{3-x}}} = \left[2x^2 + \left(x^7 - 8x(3-x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{1}{7}}$

$y' = \frac{1}{7}\left[2x^2 + \left(x^7 - 8x(3-x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{-\frac{6}{7}}\left[4x + \frac{1}{2}\left(x^7 - 8x(3-x)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(7x^6 - \left(8(3-x)^{\frac{1}{2}} + 8x\left(\frac{1}{2}\right)(3-x)^{-\frac{1}{2}}(-1)\right)\right)\right]$

Derivatives Exam Review:

4 b) $y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{(3+x^2)}$

$$y' = \frac{(3+x^2)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(2x)}{(3+x^2)^2}$$

$$y' = \frac{2\cdot\frac{3}{2}x^{-1/2} + 2\cdot\frac{1}{2}x^{3/2} - 2\cdot 2x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{3x^{-1/2} + x^{3/2} - 4x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{x^{-1/2}(3 + x^2 - 4x^2)}{2(3+x^2)^2}$$

$$y' = \frac{3-3x^2}{2x^{1/2}(3+x^2)^2} = \frac{3(1-x^2)}{2\sqrt{x}(3+x^2)^2}$$

Curve Sketching:

1. Plot all points: x-int, y-int, max, min, I.P.,

2. Plot all asymptotes (Check behaviour near VA.)

3. Use intervals of inc/dec and concavity to connect everything

Curve Sketching

Example:

Examine the function $f(x) = 3x^5 - 5x^3$ with respect to…

- Intercepts $f(x)$
- ~~Symmetry~~
- Asymptotes (No asymptotes for polynomial functions)
- Intervals of Increase or Decrease $f'(x)$
- Local Maximum and Minimum values $f(x)$
- $f''(x)$ Concavity and Points of Inflection $f(x)$
- Sketch the Curve

$f(x) = 3x^5 - 5x^3$ | $f'(x) = 15x^4 - 15x^2$ | $f''(x) = 60x^3 - 30x$

$f(x) = x^3(3x^2 - 5)$ | $f'(x) = 15x^2(x^2 - 1)$ | $f''(x) = 30x(2x^2 - 1)$

$f'(x) = 15x^2(x-1)(x+1)$

① x-int $(y=0)$

$f(x) = x^3(3x^2-5)$

$0 = x^3(3x^2-5)$

$x^3 = 0$, $x = 0$, $(0,0)$ | $3x^2 - 5 = 0$, $\frac{3x^2}{3} = \frac{5}{3}$, $x^2 = \frac{5}{3}$, $x = \pm\sqrt{5/3}$, $(1.29, 0)$ + $(-1.29, 0)$

② y-int $(x=0)$

$f(x) = 3x^5 - 5x^3$

$f(0) = 3(0)^5 - 5(0)^3$

$f(0) = 0$

$(0,0)$

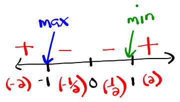
③ Intervals of Inc/Dec.

$f'(x) = 15x^2(x-1)(x+1)$

$0 = 15x^2(x-1)(x+1)$

$15x^2 = 0$, $x^2 = 0$, $x = 0$ | $x - 1 = 0$, $x = 1$ | $x + 1 = 0$, $x = -1$

CV: $x = -1, 0, 1$



Increasing on $(-\infty, -1)$ + $(1, \infty)$

Decreasing on $(-1, 0)$ + $(0, 1)$ or $(-1, 1)$

④ max @ $x = -1$

$f(x) = 3x^5 - 5x^3$

$f(-1) = 3(-1)^5 - 5(-1)^3$

$f(-1) = -3 + 5$

$f(-1) = 2$

$(-1, 2)$

⑤ min @ $x = 1$

$f(x) = 3x^5 - 5x^3$

$f(1) = 3(1)^5 - 5(1)^3$

$f(1) = 3 - 5$

$f(1) = -2$

$(1, -2)$

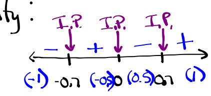
⑥ Intervals of Concavity:

$f''(x) = 30x(2x^2 - 1)$

$0 = 30x(2x^2 - 1)$

$30x = 0$, $x = 0$ | $2x^2 - 1 = 0$, $2x^2 = 1$, $x^2 = \frac{1}{2}$, $x = \pm\sqrt{1/2}$, $x \simeq \pm 0.707$

CV: $x = -0.7, 0, 0.7$



CD on $(-\infty, -0.7)$ + $(0, 0.7)$

CU on $(-0.7, 0)$ + $(0.7, \infty)$

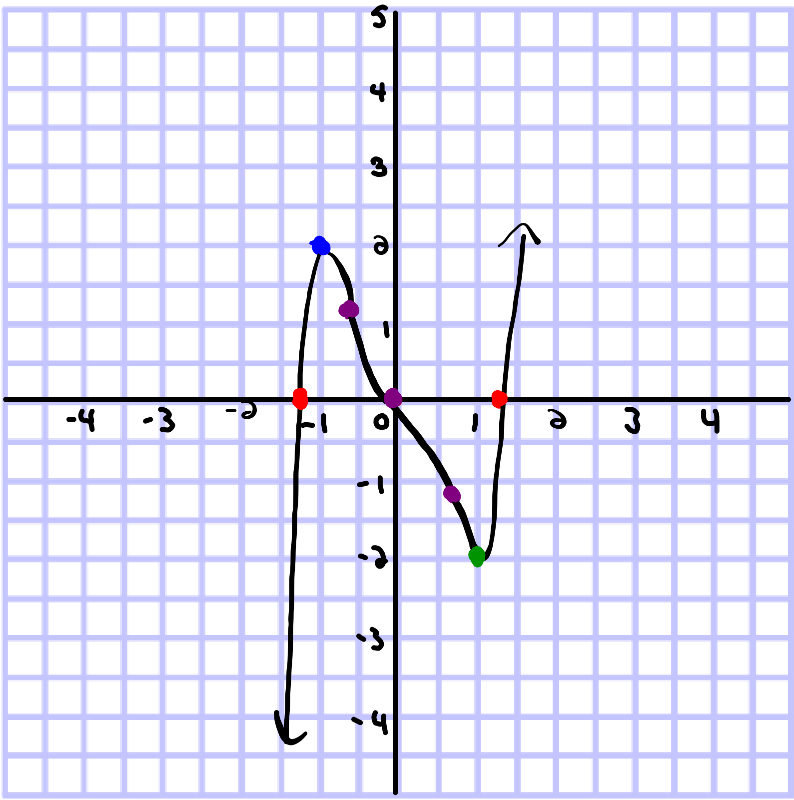
⑦ Inflection Points

$f(x) = 3x^5 - 5x^3$

$f(-0.7) = 3(-0.7)^5 - 5(-0.7)^3 = -0.504 + 1.715 = 1.2$ $(-0.7, 1.2)$

$f(0) = 3(0)^5 - 5(0)^3 = 0 - 0 = 0$ $(0,0)$

$f(0.7) = 3(0.7)^5 - 5(0.7)^3 = 0.504 - 1.715 = -1.2$ $(0.7, -1.2)$


5
4
3
2
1
-4
-3
-2
-1
0
1
2
3
4
-1
-2
-3
-4

Curve Sketching

Examine the function $f(x)=\frac{x^2}{1-x^2}$ with respect to...

$f'(x)=\frac{2x}{(1-x^2)^2}$

$f''(x)=\frac{2(1+3x^2)}{(1-x^2)^3}$

- Intercepts $f(x)$
- ~~Symmetry~~
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

① x-int $(y=0)$

$f(x)=\frac{x^2}{1-x^2}$

$0=\frac{x^2}{1-x^2}$

$0=x^2$

$0=x$

$(0,0)$

② y-int $(x=0)$

$f(x)=\frac{x^2}{1-x^2}$

$f(0)=\frac{(0)^2}{1-(0)^2}$

$f(0)=\frac{0}{1}=0$

$(0,0)$

③ Vertical Asymptote:
(zeroes of the denominator)

$f(x)=\frac{x^2}{1-x^2}$

VA: $1-x^2=0$

$(1-x)(1+x)=0$

$1-x=0 \mid 1+x=0$

$\boxed{1=x} \quad \boxed{x=-1}$

④ Horizontal Asymptote:

$\lim_{x\to\infty}\frac{x^2}{1-x^2}=\frac{1}{-1}=-1$

$\boxed{y=-1}$

⑤ Intervals of Inc/Dec.

$f'(x)=\frac{2x}{(1-x^2)^2}$

min

$-\ -\ +\ +$

$(-2)\ -1\ (-0.5)\ 0\ (0.5)\ 1\ (2)$

$2x=0$

$x=0$

$(1-x^2)^2=0$

$1-x^2=0$

$1=x^2$

$\pm1=x$

CV: $x=-1,0,1$

Increasing on $(0,\infty)$

Decreasing on $(-\infty,0)$

⑥ min @ $x=0$

$f(x)=\frac{x^2}{1-x^2}$

$f(0)=\frac{(0)^2}{1-(0)^2}=0$

$(0,0)$

⑦ Intervals of concavity

$f''(x)=\frac{2(1+3x^2)}{(1-x^2)^3}$

I.P. I.P.

$-\ +\ -$

$(-2)\ -1\ (0)\ 1\ (2)$

$2(1+3x^2)=0$

$1+3x^2=0$

$3x^2=-1$

$x^2=-\frac{1}{3}$

Not Possible

(Numerator is always positive)

$(1-x^2)^3=0$

$1-x^2=0$

$1=x^2$

$\pm1=x$

CD on $(-\infty,-1)+(1,\infty)$

CU on $(-1,1)$

⑧ Inflection Points:

when $x=-1$

$f(x)=\frac{x^2}{1-x^2}$

$f(-1)=\frac{(-1)^2}{1-(-1)^2}=\frac{1}{0}=$ und.

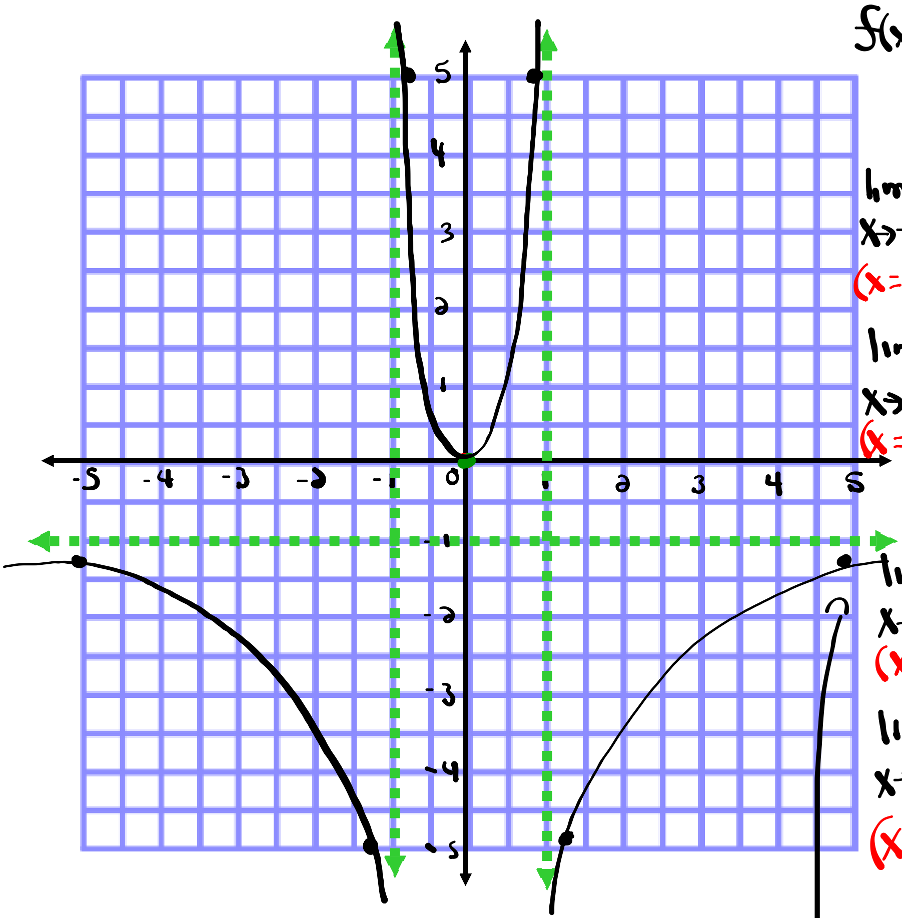
when $x=1$

$f(x)=\frac{x^2}{1-x^2}$

$f(1)=\frac{(1)^2}{1-(1)^2}=\frac{1}{0}=$ u

$$f(x) = \frac{x^2}{1-x^2}$$

$$f(x) = \frac{x^2}{(1-x)(1+x)}$$



$$\lim_{x \to -1^-} f(x) = \frac{(+)}{(+)(-)} = -\infty$$

$(x=-1.1)$

$$\lim_{x \to -1^+} f(x) = \frac{(+)}{(+)(+)} = +\infty$$

$(x=-0.9)$

$$\lim_{x \to 1^-} f(x) = \frac{(+)}{(+)(+)} = +\infty$$

$(x=0.9)$

$$\lim_{x \to 1^+} f(x) = \frac{(+)}{(-)(+)} = -\infty$$

$(x=1.1)$

$$f(5) = \frac{(5)^2}{1-(5)^2} = \frac{25}{-24}$$

Synthetic Sub:

$-27+45+6-24$

$$x^3+5x^2-2x-24$$

Find an x value that makes the polynomial equal 0

$(2)^3+5(2)^2-2(2)-24$

$8+20-4-24$

0

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -2 & -24 \\ & & 2 & 14 & 24 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

$x=2$

$(x-2)(x^2+7x+12)$ ← simple trinomial

$\underline{3}+\underline{4}=7$

$\underline{3}\times\underline{4}=12$

$(x-2)(x+3)(x+4)$

Curve Sketching:

$$f(x) = \frac{x^2+5x+6}{x^2-9} = \frac{(x+2)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}} = \frac{x+2}{x-3}$$

x int:

$x+2=0$

$x=-2$

$(-2,0)$

y int:

$f(0) = \frac{0+2}{0-3}$

$f(0) = -\frac{2}{3}$

$(0,-2/3)$

VA:

$x-3=0$

$x=3$

HA: $\lim_{x\to\infty} \frac{x+2}{x-3} = 1$

$y=1$

Hole:

$x+3=0$

$x=-3$

$f(-3) = \frac{-3+2}{-3-3}$

$f(-3) = \frac{1}{6}$