Questions from homework

Rational Functions

We will explore the properties of rational functions of this form so we can predict the locations of vertical, horizontal, and oblique asymptotes. We will also be able to identify the roots of the function and any other points of discontinuity (holes).

$$y = \frac{(x+2)(x+3)}{(x-1)}$$

$$y = \frac{(x+2)(x+3)}{(x+2)}$$

$$y = \frac{x^2+2x-3}{x^2+3x-4}$$

$$f(x) = \frac{4x}{x^2-4}$$

$$y = \frac{(x+3)(x-1)}{(x+4)(x-1)}$$

① Roots (x-int) $x = -3$

② V.A. $x = -4$

③ H.A. $y = 1$

④ Holes: $x = 1$

Roots (x-intercepts)

Are given by the zeroes of the numerator.

Vertical Asymptotes:

Are given by the zeroes of the denominator

Horizontal Asymptotes:

If the numerator and denominator have the same degree, then the horizontal asymptote is given by the quotient of the leading coefficients of the numerator and denominator.

If the degree of the denominator is greater than that of the numerator, then the horizontal asymptote is given by $y = 0$.

If the degree of the denominator is less than that of the numerator, then there is no horizontal asymptote (*an oblique asymptote exists).*

Holes

Occur when the same factor is in the numerator and the denominator.

This table shows whether a factor of a rational function results in a vertical asymptote, a root, or another point of discontinuity (hole).

Type of Factor	Vertical Asymptote	Hole	Zero of Function (Roots)
Appears in numerator only			Zero of factor
Appears in denominator only	Zero of factor		
Appears in numerator and (to equal or lesser power) denominator		Zero of common factor	
Appears in numerator and (to a greater power) denominator	Zero of common factor		

Examples

Find the roots, vertical and horizontal asymptotes, and any other points of discontinuity (hole) for the following rational functions.

$$f(x)=\frac{(x+1)(x-2)}{(x-3)(x-1)}$$

① Roots (x-int): $x=-1, 2$

② V.A.: $x=1, 3$

$$F(x)=\frac{x^2-x-2}{x^2-4x+3}$$

③ H.A.: $y=1$

④ Holes: None

$$f(x)=\frac{x^2+7x+10}{x^2-3x-4}$$

① Roots: $x=-5, -2$

② V.A.: $x=-1, 4$

$$=\frac{(x+5)(x+2)}{(x-4)(x+1)}$$

③ H.A.: $y=1$

④ Holes: None

$$y=\frac{x^2+2x-3}{x^2+3x-4}$$

① Roots: $x=-3$

② V.A.: $x=-4$

$$=\frac{(x+3)(x-1)}{(x+4)(x-1)}$$

③ H.A.: $y=1$

④ Holes: $x=1$

$$f(x)=\frac{x^2+7x+10}{x^4-5x^2+4}$$

① Roots: $x=-5$

② V.A.: $x=-1, 1, 2$

$$=\frac{(x+5)(x+2)}{(x^2-4)(x^2-1)}$$

③ H.A.: $y=0$

④ Hole: $x=-2$

$$=\frac{(x+5)(x+2)}{(x-2)(x+2)(x-1)(x+1)}$$

$$f(x) = \frac{2x^2+6x}{x^2-9}$$

① Roots (x int)

$x = 0$

② V.A.

$x = 3$

$$= \frac{2x(x+3)}{(x-3)(x+3)}$$

③ H.A.

$y = 2$

④ Holes

$x = -3$

$$= \frac{2x}{(x-3)}$$

Homework