

Convert the following complex numbers from *rectangular* to *polar form*

$$\underline{(5\sqrt{3} - 5i)}$$

$$\left. \begin{array}{l} a = 5\sqrt{3} \\ b = -5 \end{array} \right\} \underline{\text{Quad 4}}$$

$$\begin{aligned} \textcircled{1} r &= \sqrt{(5\sqrt{3})^2 + (-5)^2} & \textcircled{2} \alpha &= \tan^{-1}\left(\frac{5}{5\sqrt{3}}\right) \\ r &= \sqrt{75 + 25} & \alpha &= 30^\circ \\ r &= 10 \end{aligned}$$

$$\begin{aligned} \textcircled{3} & \text{Quad 4} \\ \theta &= 360 - \alpha \\ \theta &= 360^\circ - 30^\circ \\ \theta &= 330^\circ \end{aligned}$$

$$\textcircled{4} \boxed{10 \text{cis } 330^\circ}$$

$$\underline{(1 - i\sqrt{3})}$$

$$\left. \begin{array}{l} a = 1 \\ b = -\sqrt{3} \end{array} \right\} \text{Quad 4}$$

$$\begin{aligned} \textcircled{1} r &= \sqrt{(1)^2 + (\sqrt{3})^2} & \textcircled{2} \alpha &= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) \\ r &= \sqrt{1 + 3} & \alpha &= 60^\circ \\ r &= 2 \end{aligned}$$

$$\begin{aligned} \textcircled{3} & \text{Quad 4} \\ \theta &= 360 - \alpha \\ \theta &= 360 - 60 \\ \theta &= 300^\circ \end{aligned}$$

$$\textcircled{4} \boxed{2 \text{cis } 300^\circ}$$

**Multiply your answers**

$$(10 \text{cis } 330^\circ)(2 \text{cis } 300^\circ)$$

$$20 \text{cis } 630^\circ$$

$$\boxed{20 \text{cis } 270^\circ} \text{ Polar}$$

$$\boxed{0 - 20i} \text{ Rectangular}$$

**Divide your answers**

$$\frac{10 \text{cis } 330^\circ}{2 \text{cis } 300^\circ}$$

$$\boxed{5 \text{cis } 30^\circ} \text{ Polar}$$

$$\boxed{4.33 + 2.5i} \text{ Rec.}$$

Questions from Homework

$$1c) \frac{(4-4i\sqrt{3})(2\sqrt{3}+2i)(1+i)}{(5-5i)(-\sqrt{3}+i)}$$

$$\begin{array}{llll} a=4 & r=\sqrt{16+48} & \alpha=\tan^{-1}\left(\frac{4\sqrt{3}}{4}\right) & \text{Quad 4} \\ b=-4\sqrt{3} & r=\sqrt{64} & \alpha=60^\circ & \theta=300^\circ \\ & r=8 & & \end{array}$$

$$\begin{array}{llll} a=2\sqrt{3} & r=\sqrt{12+4} & \alpha=\tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) & \text{Quad 1} \\ b=2 & r=4 & \alpha=30^\circ & \theta=30^\circ \end{array}$$

$$\begin{array}{llll} a=1 & r=\sqrt{1+1} & \alpha=\tan^{-1}\left(\frac{1}{1}\right) & \text{Quad 1} \\ b=1 & r=\sqrt{2} & \alpha=45^\circ & \theta=45^\circ \end{array}$$

$$\begin{array}{llll} a=5 & r=\sqrt{25+25} & \alpha=\tan^{-1}\left(\frac{5}{5}\right) & \text{Quad 4} \\ b=-5 & r=\sqrt{50} & \alpha=45^\circ & \theta=315^\circ \\ & r=5\sqrt{2} & & \end{array}$$

$$\begin{array}{llll} a=-\sqrt{3} & r=\sqrt{3+1} & \alpha=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) & \text{Quad 2} \\ b=1 & r=2 & \alpha=30^\circ & \theta=150^\circ \end{array}$$

$$\frac{(8 \text{ cis } 300^\circ)(4 \text{ cis } 30^\circ)(\sqrt{2} \text{ cis } 45^\circ)}{(5\sqrt{2} \text{ cis } 315^\circ)(2 \text{ cis } 150^\circ)}$$

$$\frac{32\sqrt{2} \text{ cis } 375^\circ}{10\sqrt{2} \text{ cis } 465^\circ}$$

$$3.2 \text{ cis } (-90^\circ)$$

$$\boxed{3.2 \text{ cis } 270^\circ} \text{ Polar}$$

$$3.2 \cos 270^\circ + 3.2i \sin 270^\circ$$

$$3.2(0) + 3.2i(-1)$$

$$\boxed{-3.2i} \text{ Rectangular}$$

You may have noticed a shortcut when multiplying and dividing complex numbers in Polar form.

**For Multiplication**

**Multiply the "r" values and add the arguments.**

**For Division**

**Divide the "r" values and subtract the arguments.**

## De Moivre's Theorem

$$(rcis\theta)^n = r^n cisn\theta$$

Evaluate

$$(2\cos\frac{\pi}{6} + 2i\sin\frac{\pi}{6})^5$$

$$(2cis\frac{\pi}{6})^5$$

$$2^5 cis(5 \cdot \frac{\pi}{6})$$

$$\boxed{32cis\frac{5\pi}{6}}$$

In Degrees:

$$\frac{\pi}{6} \cdot \frac{180}{\pi} = \underline{30^\circ}$$

$$(2cis\underline{30^\circ})^5$$

$$2^5 cis(5 \cdot 30)$$

$$\boxed{32cis150^\circ}$$

Remember  $(rcis\theta)^n = r^n cis n\theta$

$$(3 - 4i)^5$$

$$\left. \begin{array}{l} a = 3 \\ b = -4 \end{array} \right\} \text{Quad 4}$$

$$\begin{aligned} \textcircled{1} r &= \sqrt{(3)^2 + (-4)^2} \\ r &= \sqrt{9 + 16} \\ r &= 5 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \alpha &= \tan^{-1}\left(\frac{4}{3}\right) \\ \alpha &= 53^\circ \end{aligned}$$

$$\begin{aligned} \textcircled{3} \text{Quad 4} \\ \theta &= 360 - \alpha \\ \theta &= 360 - 53^\circ \\ \theta &= 307^\circ \end{aligned}$$

$$\textcircled{4} \boxed{5 cis 307^\circ}$$

$$(5 cis 307^\circ)^5$$

$$5^5 cis (5 \cdot 307^\circ)$$

$$3125 cis 1535^\circ$$

$$\boxed{3125 cis 95^\circ} \text{ Polar}$$

$$\boxed{-272.4 + 3113.1i} \text{ Rectangular}$$

