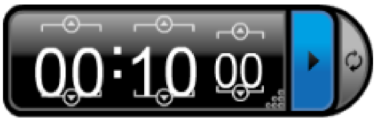
# Warm Up




1. Simplify: $$\frac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$$

2. Factor each of the following:

$x^{27}-1$ $\qquad$ $(x^2+1)^{\frac{1}{2}}+3(x^2+1)^{-\frac{1}{2}}$

a) $(x^9-1)(x^{18}+x^9+1)$

$(x^3-1)(x^6+x^3+1)(x^{18}+x^9+1)$

$(x-1)(x^2+x+1)(x^6+x^3+1)(x^{18}+x^9+1)$

b) $(x^2+1)^{1/2}+3(x^2+1)^{-1/2}$

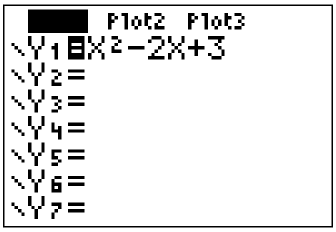
$(x^2+1)^{-1/2}\left[(x^2+1)^1+3\right]$

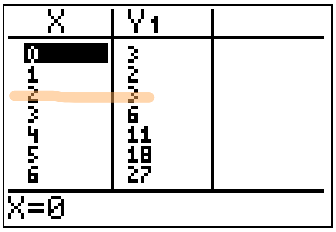
$(x^2+1)^{-1/2}(x^2+4)$

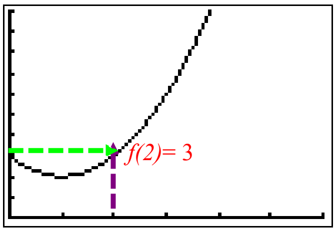
↳ $\dfrac{x^2+4}{\sqrt{x^2+1}}$

# Limit of a Function

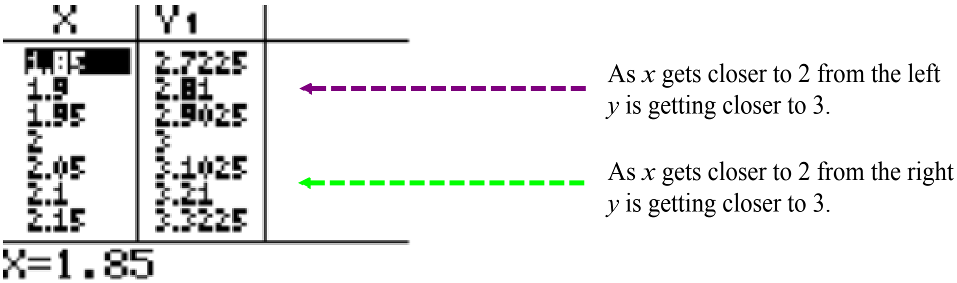
Let's examine the function $f(x) = x^2 - 2x + 3$



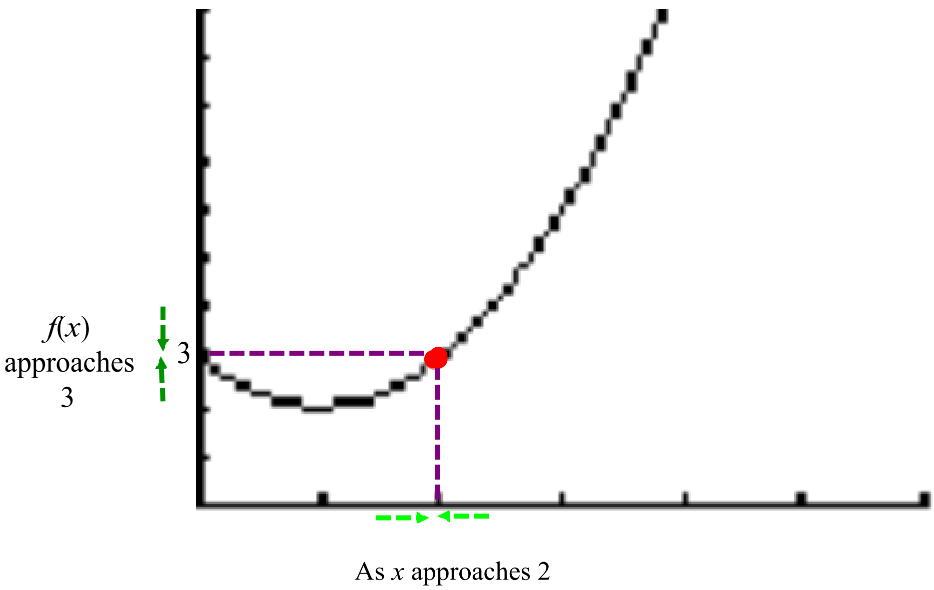




We can see that *f(2)*= 3...let's check the behaviour of *f* as we get closer and closer to $x = 2$.




From the above, the notion of the limit of a function arises...




Notation: $\lim_{x \to 2} f(x) = 3$

"The limit of the function *f*(*x*) as *x* approaches 2 is equal to 3."

## The common sense definition of a limit...

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## A formal definition of a limit...

We write $\lim_{x \to a} f(x) = L$ if we can make the values of $f(x)$ <u>arbitrarily close to $L$</u>

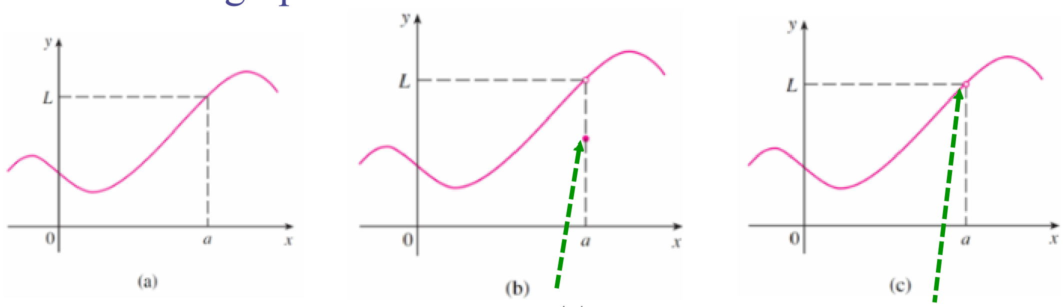
- (as close to $L$ as we like)

by taking $x$ to be <u>sufficiently close to $a$</u>

- (on either side of $a$)

but <u>not</u> equal to $a$.

Look at the graphs of these three functions...




Notice $f(a) \neq L$

Notice $f(a)$ is undefined

But in each case, regardless of what happens at $a$, it is true that

$$\lim_{x \to a} f(x) = L$$

# Evaluating Limits

## I. Using a Graph:

- We looked at this in the previous two examples

## II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x \to -2} \frac{x^2 - 2x + 1}{x + 3}$$

$$\lim_{x \to 3} \left(16 - x^2\right)$$

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$

  $\Rightarrow$ Factor
  $\Rightarrow$ Rationalize
  $\Rightarrow$ Expand
  $\Rightarrow$ Find Common Denominators

Examples:

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

$$\lim_{h \to 0} \frac{\sqrt{4 + h} - 2}{h}$$

# Questions from Homework

Try these...remember to use your algebra skills to try and eliminate the **indeterminate form**.

Direct Substitution leads to $\frac{0}{0}$

$$\lim_{x\to2}\frac{(x+2)^2-16}{x^2-4}$$

$$\lim_{x\to2}\frac{(x+2+4)(x+2-4)}{(x+2)(x-2)}$$

$$\lim_{x\to2}\frac{(x+6)(x-2)}{(x+2)(x-2)}$$

$$\lim_{x\to2}\frac{(2+6)}{(2+2)}=\frac{8}{4}=\boxed{2}$$

$$\lim_{x\to-2}\frac{x^4-16}{x^3+8}$$

$$\lim_{x\to-2}\frac{(x^2+4)(x^2-4)}{(x+2)(x^2-2x+4)}$$

$$\lim_{x\to-2}\frac{(x^2+4)(x+2)(x-2)}{(x+2)(x^2-2x+4)}$$

$$\lim_{x\to2}\frac{((-2)^2+4)(-2-2)}{((-2)^2-2(-2)+4)}=\frac{8(-4)}{(4+4+4)}$$

$$=\frac{-32}{12}=\boxed{\frac{-8}{3}}$$

$$\lim_{x\to0}\frac{x^2+3x}{(x+2)^2-(x-2)^2}$$

$$\lim_{x\to0}\frac{x(x+3)}{((x+2)-(x-2))((x+2)+(x-2))}$$

$$\lim_{x\to0}\frac{x(x+3)}{(x+2-x+2)(x+2+x-2)}$$

$$\lim_{x\to0}\frac{x(x+3)}{(4)(2x)}$$

$$\lim_{x\to0}\frac{x(x+3)}{8x}=\boxed{\frac{3}{8}}$$

$$\lim_{x\to2}\frac{(2x)\left(\frac{1}{x}-\frac{1}{2}\right)(2x)}{(x-2)(2x)}$$

$$\lim_{x\to2}\frac{(2-x)}{2x(x-2)}\quad(2-x \to -1)$$

$$\lim_{x\to2}\frac{-1}{2x}=\frac{-1}{2(2)}=\boxed{\frac{-1}{4}}$$

# Homework

# 2 → Direct Substitution
# 3 → Direct Substitution
# 4/5 → Indeterminate form
# 6 → Mixed