

Warm up

As it aged, a maple tree produced sap according to the pattern shown in the table below.

Year	2001	2002	2003	2004
Sap (Litres)	$t_1 = 60.000$	$t_2 = 57.000$	$t_3 = 54.150$	$t_4 = 51.4425$

a) Does the data follow an arithmetic or geometric pattern?

$$r = \frac{t_2}{t_1} = \frac{57}{60} = 0.95$$

b) Write down a formula for t_n ? $t_n = ar^{n-1}$

$$a = 60$$

$$r = 0.95$$

$$t_n = 60(0.95)^{n-1}$$

c) Assuming the pattern continues, how long will it take for the sap production to be approximately 17.5L?

$$t_n = 17.5$$

$$a = 60$$

$$r = 0.95$$

$$n = ?$$

$$t_n = ar^{n-1}$$

$$\frac{17.5}{60} = \frac{60(0.95)^{n-1}}{60}$$

$$0.291\bar{6} = (0.95)^{n-1}$$

$$(0.95)^{24} = (0.95)^{n-1}$$

$$24 = n - 1$$

$$25 = n$$

In 25 years
the tree will
produce 17.5L

Questions from homework

$$\textcircled{9} \quad 1 + 4 + 9 + 16$$

$$\sum_{n=1}^4 n^2$$

$$= (1)^2 + (2)^2 + (3)^2 + (4)^2$$

$$= 1 + 4 + 9 + 16$$

$$= 30$$

$$\textcircled{10} \quad 3 + 6 + 12 + 24 + 48$$

$$a = 3$$

$$r = 2$$

$$t_n = (3)(2)^{n-1}$$

$$\sum_{n=1}^5 3(2)^{n-1}$$

Limit (of a sequence - t_n)

A finite number L that the value of t_n gets closer and closer to, or "approaches," as n becomes very large, or "approaches infinity."

The value of t_n can be made as close as you like to L by using a sufficiently large value for n .

The notation for a limit is

$$\lim_{n \rightarrow \infty} t_n = L$$

Converging Sequence \rightarrow Has a Limit

A sequence in which the terms approach a limit

For example, $\frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}, \dots$ converges to 1

0.25, 0.40, 0.50, 0.57, ...

The above sequence was generated using the following general term.

$$t_n = \frac{n}{n+3}$$

What happens if "n" is a very large number?

$$t_{100} = \frac{100}{103} = 0.97$$

$$t_{1000} = \frac{1000}{1003} = 0.997$$

$$\lim_{n \rightarrow \infty} \frac{n}{n+3} = 1$$

Diverging Sequence \rightarrow Has no Limit

A sequence in which the terms do not approach a limit

For example, 1, 2, 3, 4, ... diverges. (no limit exists)

The above sequence was generated using the following general term.

$$t_n = n$$

What happens if "n" is a very large number?

$$t_{100} = 100$$

$$t_{9999} = 9999$$

$$\lim_{n \rightarrow \infty} n = \infty \rightarrow \text{Limit does not exist}$$

No Limit
DNE

Decide whether each sequence *converges* or *diverges* then state the limit.

2, 4, 8, 16, 32, ... *diverges*

$$\begin{array}{l} a=2 \\ r=2 \end{array} \quad \begin{array}{l} t_n = (2)^1 (2)^{n-1} \\ t_n = (2)^n \end{array} \quad \lim_{n \rightarrow \infty} 2^n = \infty \rightarrow \text{DNE}$$

3, 1.5, 0.75, 0.375, ... *converges*

$$\begin{array}{l} a=3 \\ r=1/2 \end{array} \quad \begin{array}{l} t_n = (3) \left(\frac{1}{2}\right)^{n-1} \\ t_n = (3) \left(\frac{1}{2}\right)^n \end{array} \quad \lim_{n \rightarrow \infty} (3) \left(\frac{1}{2}\right)^{n-1} = 0$$

$$\begin{aligned} t_{10} &= 3 \left(\frac{1}{2}\right)^9 \\ &= 0.0058 \end{aligned}$$

Infinite Sequences

Suppose we have a sequence defined by $t_n = \frac{n}{2n+1}$, $n \in \mathbb{N}$

Generate the first 4 terms of the sequence

$$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \dots \frac{100}{201}$$

You may notice that as " n " increases " t_n " approaches $\frac{1}{2}$

Symbolically this is written $\lim_{n \rightarrow \infty} \frac{n}{2n+1} = \frac{1}{2}$

and is read "The limit as n approaches infinity of n over $(2n+1)$ is $\frac{1}{2}$."

Algebraically we solve by dividing the numerator and the denominator by the highest power of n . (degree)

$$\frac{n}{2n+1}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{n}{n}}{\frac{2n}{n} + \frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} \frac{1}{2 + \frac{1}{n}} \longrightarrow 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{2+0} = \frac{1}{2}$$

Find the limit if it exists

Diverging

$$t_n = \frac{n+5}{1}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{n}{n} + \frac{5}{n}}{\frac{1}{n}} \rightarrow \frac{1+0}{0}$$

$$\lim_{n \rightarrow \infty} \frac{1+0}{0}$$

$$\lim_{n \rightarrow \infty} \frac{1}{0} = \text{DNE}$$

Converging

$$t_n = \frac{3n+1}{4n-2}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{3n}{n} + \frac{1}{n}}{\frac{4n}{n} - \frac{2}{n}}$$

$$\lim_{n \rightarrow \infty} \frac{3+0}{4-0} = \frac{3}{4}$$

if the degree of the numerator and denominator are the same, then your limit will be the quotient of the leading coefficients.

$$\lim_{n \rightarrow \infty} \frac{n^2 - 2}{4 + 3n^2} = \frac{1}{3}$$

if the degree of the numerator is larger than the degree of the denominator, then your limit will not exist.

$$\lim_{n \rightarrow \infty} \frac{n^2 + 2n}{n - 3} = \text{DNE}$$

if the degree of the denominator is larger than the degree of the numerator, then your limit will always equal 0.

$$\lim_{n \rightarrow \infty} \frac{1}{3n^5 - 2} = 0$$

Homework

#1 b)

#2

#3

#4

$$t_n = \frac{1 - n^2}{n^3 + 2}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{n^3} - \frac{1}{n^3}}{\frac{n^3}{n^3} + \frac{2}{n^3}}$$

$$\lim_{n \rightarrow \infty} \frac{\boxed{\frac{1}{n^3}} - \boxed{\frac{1}{n^3}}}{1 + \boxed{\frac{2}{n^3}}}$$

$$\lim_{n \rightarrow \infty} \frac{0 - 0}{1 + 0} = 0$$