

Warm Up

Solve the following system of equations:

$$4x + 9 = 3y - 6z$$

$$3z = 10 + 2x + 4y$$

$$2y = 4z - 11 - 3x$$

$$4x - 3y + 6z = -9$$

$$-2x - 4y + 3z = 10$$

$$3x + 2y - 4z = -11$$

$$4x - 3y + 6z = -9$$

$$-6x - 12y + 9z = 30$$

$$\textcircled{A} \quad \frac{-4x - 8y + 6z = 20}{-11y + 12z = 11}$$

$$\textcircled{B} \quad \frac{6x + 4y - 8z = -22}{-8y + z = 8}$$

"2x2 system"

$$-11y + 12z = 11$$

$$\Leftrightarrow \frac{-96y + 12z = 96}{85y = -85}$$

$$85y = -85$$

$$y = -1$$

$$-8y + z = 8$$

$$-8(-1) + z = 8$$

$$8 + z = 8$$

$$z = 0$$

$$4x - 3y + 6z = -9$$

$$4x - 3(-1) + 6(0) = -9$$

$$4x + 3 + 0 = -9$$

$$4x = -12$$

$$x = -3$$

$$(-3, -1, 0)$$

Questions from Homework

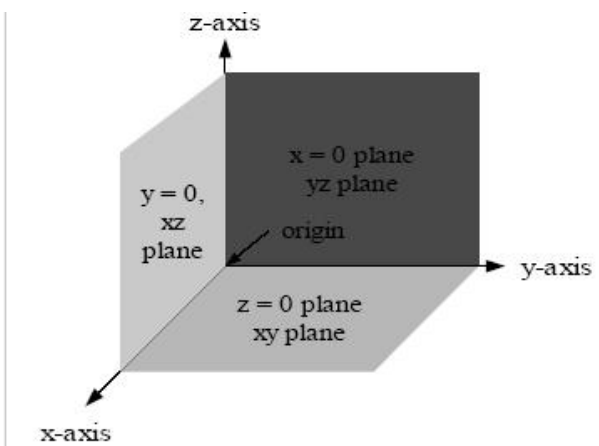
$$\begin{array}{l} \textcircled{a)} \quad 3x - 4y + 5z = 26 \\ \quad \quad 6x - 2y - 3z = -39 \\ \quad \quad x + 3y - 4z = -31 \end{array} \quad \begin{array}{l} 6x - 8y + 10z = 52 \\ \Leftrightarrow 6x - 2y - 3z = -39 \\ \hline -6y + 13z = 91 \end{array} \quad \begin{array}{l} 6x - 2y - 3z = -39 \\ \Leftrightarrow 6x + 18y - 24z = -186 \\ \hline -20y + 21z = 147 \end{array}$$

$$\begin{array}{l} -60y + 130z = 910 \\ \Leftrightarrow \frac{-60y + 63z = 441}{67z = 469} \\ \quad \quad \quad \boxed{z = 7} \end{array} \quad \begin{array}{l} -6y + 13z = 91 \\ -6y + 13(7) = 91 \\ -6y + 91 = 91 \\ -6y = 0 \\ \quad \quad \quad \boxed{y = 0} \end{array} \quad \begin{array}{l} x + 3y - 4z = -31 \\ x + 3(0) - 4(7) = -31 \\ x + 0 - 28 = -31 \\ \quad \quad \quad \boxed{x = -3} \end{array}$$

$$(-3, 0, 7)$$

ALGEBRA OF 3-SPACE

- Coordinate geometry that represents space in **three** dimensions
- Coordinates are in the form of an ordered triplet **(x, y, z)**
- Three planes exist: **xy** plane, **xz** plane, **yz** plane

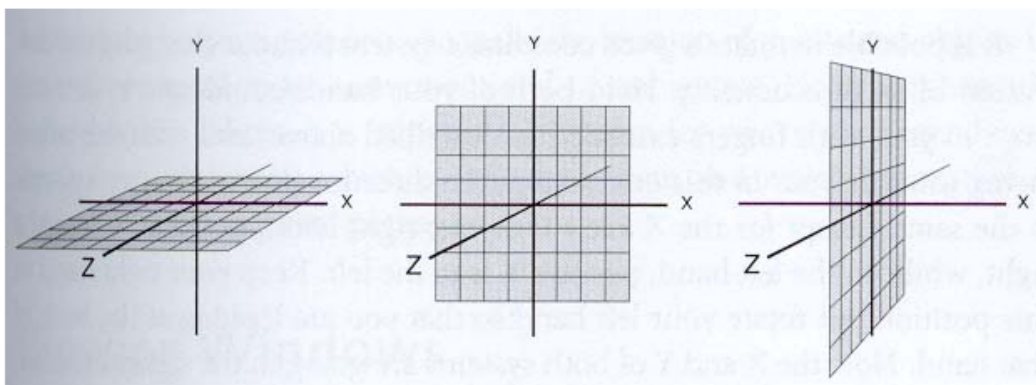


x axis -axis coming "out of the page"
y axis - horizontal axis
z axis - vertical axis

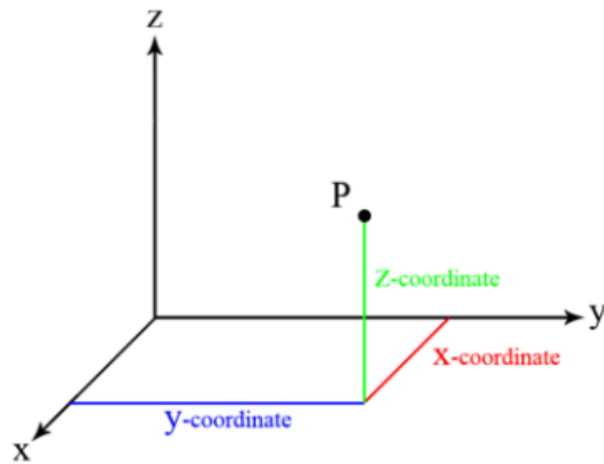
xz plane

xy plane

yz plane



Plotting Points in 3-Space

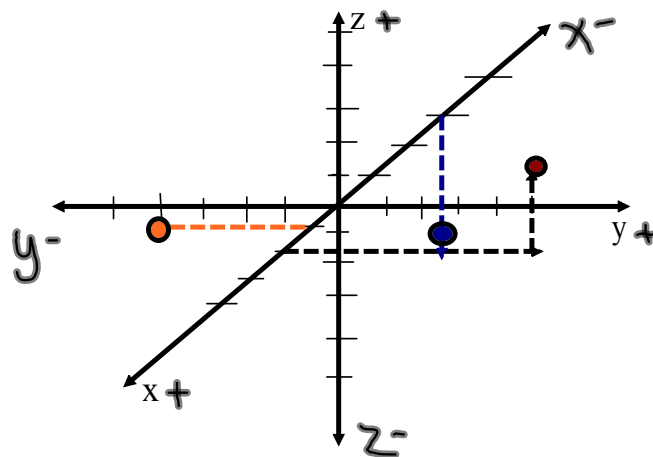


Plotting points in 3-space...

Ex: a) $(2, 6, 3)$

b) $(-3, 0, -4)$

c) $(1, -4, 0)$

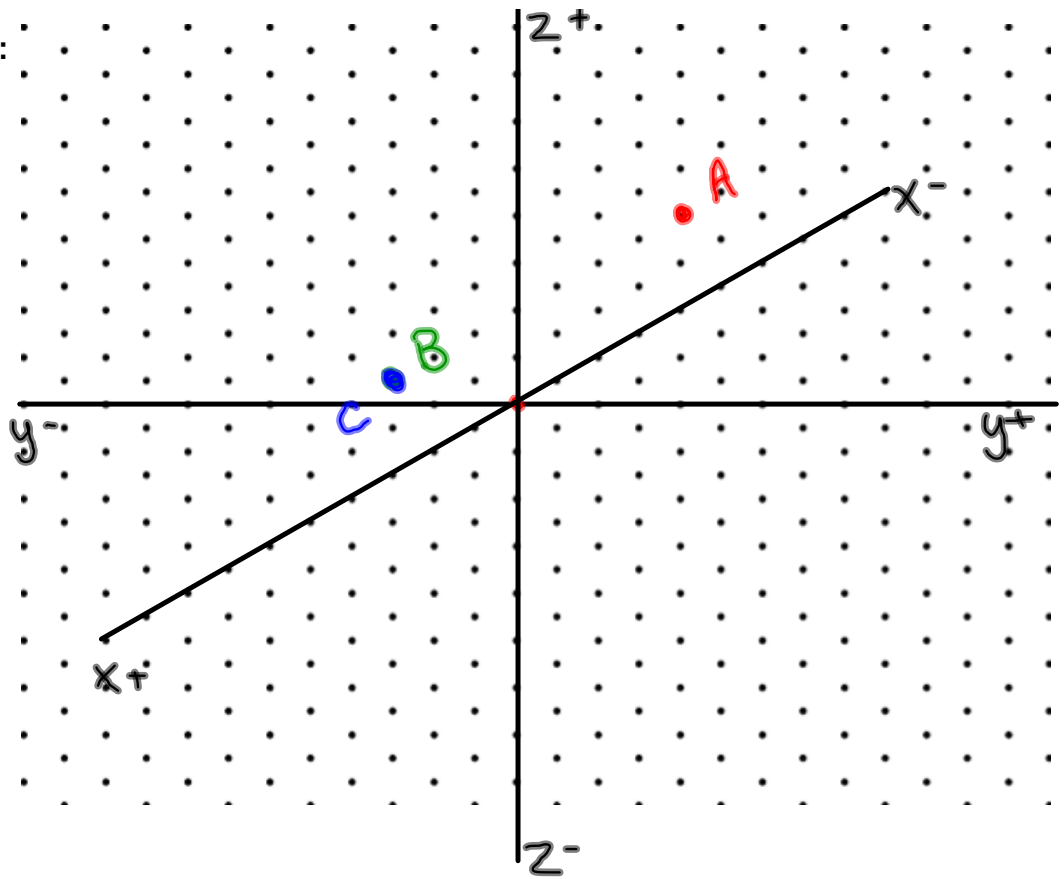


Plot the following:

A (-2, 1, 3) ●

B (3, 0, 2) ●

C (-1, -2, 0) ●



Finding Intercepts in 3D

As in two dimensions...

x intercept can be found when $y = 0$ and $z = 0$

$$(x,y,z) \longrightarrow (x,y,0)$$

y intercept can be found when $x = 0$ and $z = 0$

$$(x,y,z) \longrightarrow (0,y,0)$$

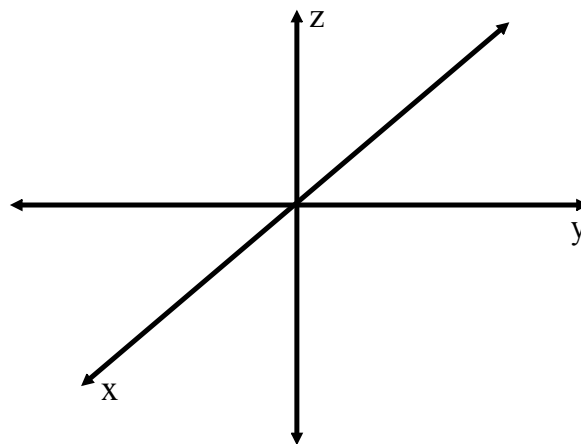
z intercept can be found when $x = 0$ and $y = 0$

$$(x,y,z) \longrightarrow (0,0,z)$$

Plotting Planes in 3-Space

- Use the **intercept method** to plot the x, y, and z intercepts to form a triangle
- The triangle is part of the plane being sketched

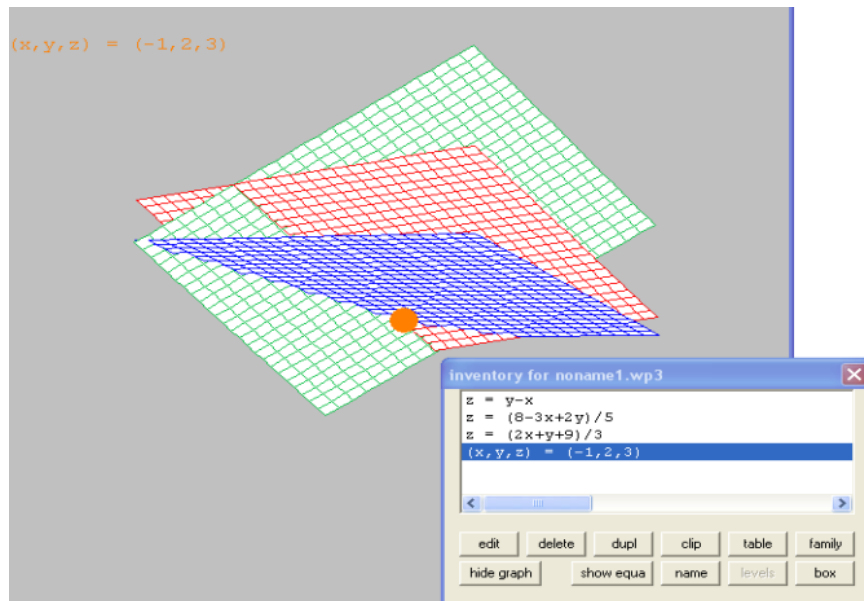
Ex. $2x - y + 3z = 6$



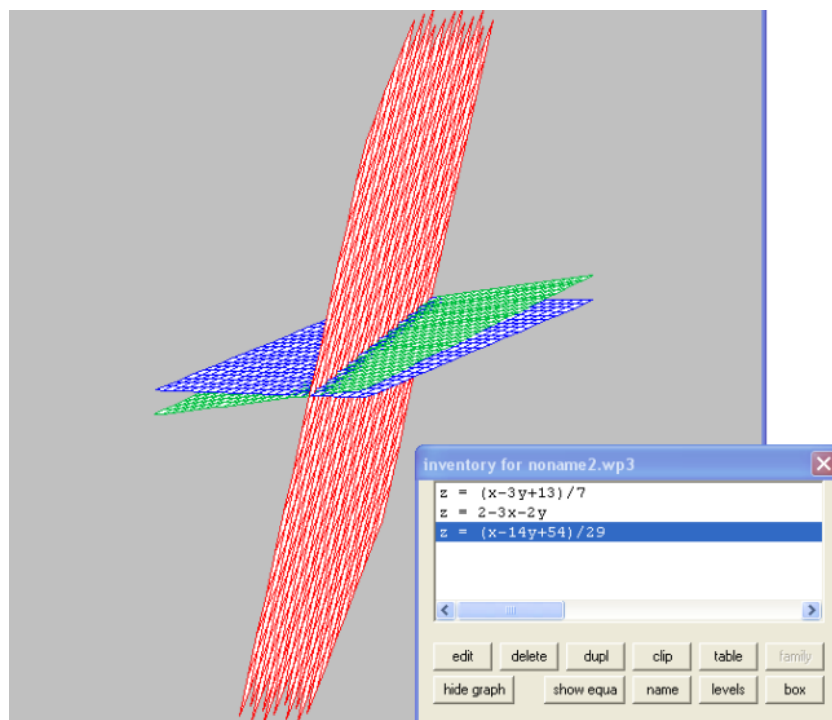
Types of Systems

Remember: Looking at **intersecting planes!**

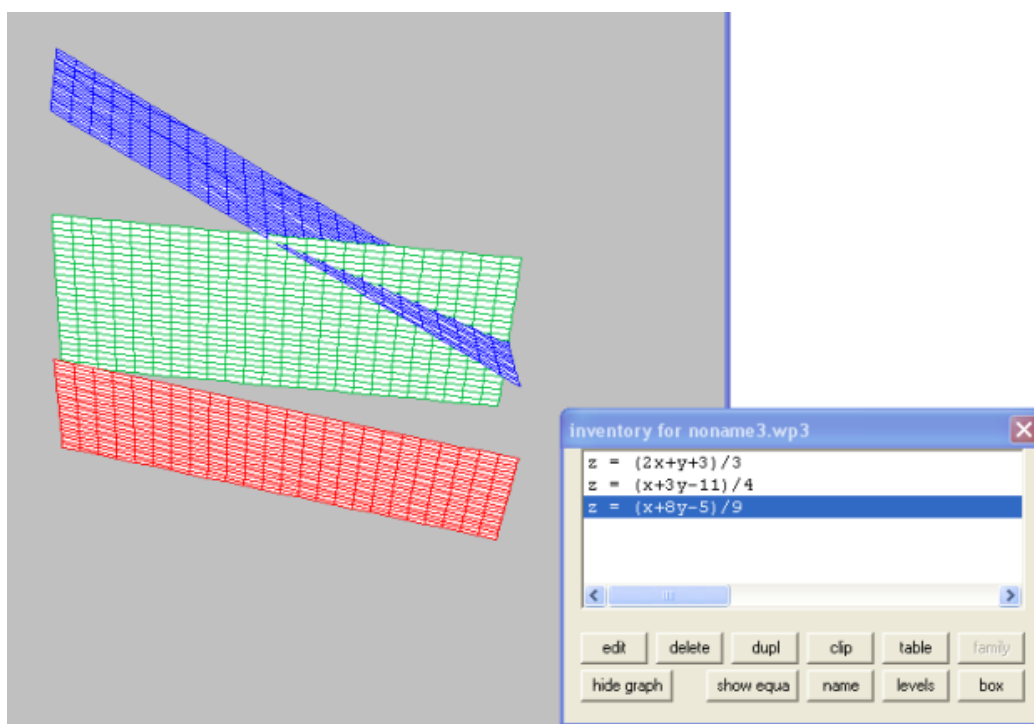
Consistent: At least one solution



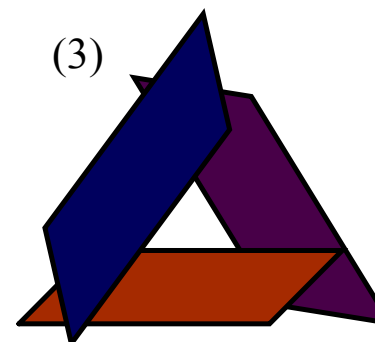
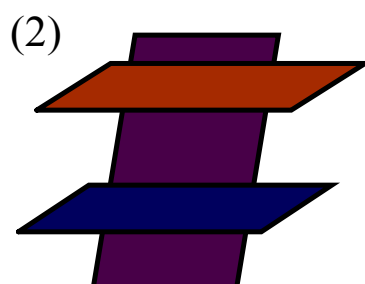
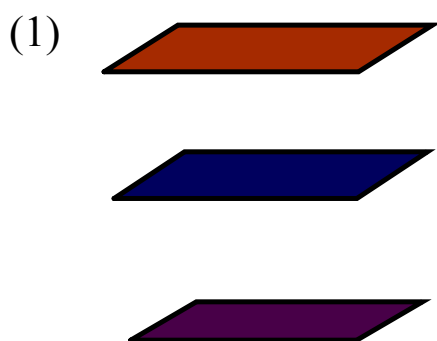
Dependent: Infinite number of solutions



Inconsistent: No Solutions



3 Possible Orientations That Give No Solution...



I. Consistent System with a Unique Solution

$$x - y + z = 0$$

$$3x - 2y + 5z = 8$$

$$2x + y - 3z = -9$$

STEPS:

- 1) Eliminate one of the variables
- 2) Solve the 2 x 2 system
- 3) Use "backward substitution" to obtain a solution

Homework

Handout: Solving Systems of Equations in 3-Space

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II. Consistent System with a Dependent Solution (must create a parametric solution)

$$x - 3y - 7z = -13$$

$$3x + 2y + z = 2$$

$$x - 14y - 29z = -54$$

Write a general solution in terms of a parameter (i.e. $z = t$). For each value assigned to the parameter there will be one distinct solution.

