

Final Review

$$\begin{aligned}
 \textcircled{1} \text{ a) } f'(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h-5) - (x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})} \\
 &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h-5} + \sqrt{x-5})} \\
 &= \frac{1}{2\sqrt{x-5}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } f'(x) &= \lim_{h \rightarrow 0} \frac{2x+2h-2}{x+h+3} - \left(\frac{2x-2}{x+3} \right) \\
 &= \lim_{h \rightarrow 0} \frac{2x^2 + 2xh - 2x + 6x + 6h - 6 - (2x^2 + 2xh + 6x - 2x - 2h - 6)}{(x+h+3)(x+3)} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + 6x + \cancel{6h} - 6 - \cancel{2x^2} - \cancel{2xh} - 6x + \cancel{2x} + \cancel{2h} + 6}{(x+h+3)(x+3)} \\
 &= \lim_{h \rightarrow 0} \frac{8h}{(x+h+3)(x+3)} = \frac{1}{x+3} = \boxed{\frac{8}{(x+3)^2}}
 \end{aligned}$$

$$\textcircled{2} \text{ a) } \boxed{f'(x) = 6x + 5}$$

$$\text{b) } f(x) = 3x^{1/2}$$

$$f'(x) = -3x^{-3/2}$$

$$2$$

$$\boxed{f'(x) = \frac{-3}{2\sqrt{x^3}}}$$

$$\text{c) } f(x) = 2x^4 + x^{1/2}$$

$$\boxed{f'(x) = 8x^3 + \frac{1}{2\sqrt{x}}}$$

$$\text{d) } f(x) = x^{2/3}$$

$$f'(x) = \frac{2}{3}x^{-1/3}$$

$$\boxed{f'(x) = \frac{2}{3\sqrt[3]{x}}}$$

$$\begin{aligned} \textcircled{3} \text{ a) } y &= (3x^2 - 2)(4x + 5) \\ y' &= (3x^2 - 2)(4) + (6x)(4x + 5) \\ y' &= 12x^2 - 8 + 24x^2 + 30x \\ y' &= 36x^2 + 30x - 8 \end{aligned}$$

$$\begin{aligned} \text{b) } g(x) &= (x^2 - 5x + 2)(4x + 1) \\ g'(x) &= (x^2 - 5x + 2)(4) + (2x - 5)(4x + 1) \\ g'(x) &= 4x^2 - 20x + 8 + 8x^2 - 18x - 5 \\ g'(x) &= 12x^2 - 38x + 3 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \text{ a) } f(x) &= \frac{2x^2 + 3}{3x - 2} \\ f'(x) &= \frac{(3x - 2)(4x) - (2x^2 + 3)(3)}{(3x - 2)^2} \\ f'(x) &= \frac{12x^2 - 8x - 6x^2 - 9}{(3x - 2)^2} \\ f'(x) &= \frac{6x^2 - 8x - 9}{(3x - 2)^2} \end{aligned}$$

$$\begin{aligned} \text{b) } y &= \frac{\sqrt{x}}{3+x^2} \\ y' &= \frac{(3+x^2)\left(\frac{1}{2}x^{-1/2}\right) - (x^{1/2})(2x)}{(3+x^2)^2} \\ y' &= \frac{25x \cdot 3 + x^2 - 2x^{3/2} \cdot 25x}{2\sqrt{x} (3+x^2)^2} \\ y' &= \frac{3+x^2 - 4x^2}{2\sqrt{x} (3+x^2)^2} \\ y' &= \frac{3-3x^2}{2\sqrt{x} (3+x^2)^2} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad y &= (x^2 - 3)^8 & \text{at } x = 2 & \quad m = 32 \\ y &= (2^2 - 3)^8 & y = 1 & \quad \rightarrow (2, 1) \\ y &= (4 - 3)^8 \\ y &= 1 \end{aligned}$$

$$\begin{aligned} y - 1 &= 32(x - 2) \\ y - 1 &= 32x - 64 \\ 0 &= 32x - y - 63 \end{aligned}$$

$$\begin{aligned} y' &= 8(x^2 - 3)^7 (2x) \\ y' &= 16x(x^2 - 3)^7 \\ m = y' &= 16(2)(4 - 3)^7 \\ m &= 32 \end{aligned}$$

