Functions Toolkit #2
Solutions

1) a) $\frac{4}{(x-6)(x+5)} - \frac{2}{(x+5)(x+3)}$

$\frac{4x+12-2x+12}{(x-6)(x+5)(x+3)}$

$\boxed{\frac{2x+24}{(x-6)(x+5)(x+3)}, \quad x \neq -5, -3, 6}$

b) $\frac{2x}{3x+5} + \frac{x}{3x^2-6x+5x-10}$

$\frac{2x}{(3x+5)} + \frac{x}{(3x+5)(x-2)}$

$\frac{2x^2-4x+x}{(3x+5)(x-2)}$

$\boxed{\frac{2x^2-3x}{(3x+5)(x-2)}, \quad x \neq -\frac{5}{3}, 2}$

c) $\frac{3\cancel{(x+2)}}{x^2} \times \frac{\cancel{x}}{\cancel{x(x+2)}}$

$\boxed{\frac{3}{x^2}, \quad x \neq -2, 0}$

d) $\frac{xy \cdot \frac{2}{x} + \frac{3}{xy} \cdot xy}{xy \cdot \frac{2}{xy} + \frac{3}{y} \cdot xy} \rightarrow \frac{2y+3}{2+3x}$

$\Rightarrow \boxed{\frac{2y+3}{3x+2} \quad x \neq -\frac{2}{3}, 0 \quad y \neq 0}$

2) a) $\frac{3}{x-2} + \frac{6}{(x-2)(x-3)} = \frac{4}{x-3}$

$3(x-3) + 6 = 4(x-2)$

$3x-9+6 = 4x-8$

$-x = -5$

$\boxed{x = 5}$

$x = 5$ is a solution

b) $\frac{x+6}{(x+2)(x-2)} = \frac{2}{x-2} + \frac{x}{x+2}$

$x+6 = 2(x+2) + x(x-2)$

$x+6 = 2x+4+x^2-2x$

$0 = x^2-x-2$

$0 = (x-2)(x+1)$

$x = 2 \quad | \quad \boxed{x = -1}$

$x = -1$ is a solution

② c) $\left(\sqrt{3x+15}\right)^2 = \left(1+\sqrt{18+x}\right)^2$

$3x+15 = 1 + 2\sqrt{18+x} + 18+x$

$2x-4 = 2\sqrt{18+x}$

$2(x-2) = 2\sqrt{18+x}$

$(x-2)^2 = \left(\sqrt{18+x}\right)^2$

$x^2-4x+4 = 18+x$

$x^2-5x-14 = 0$

$(x-7)(x+2) = 0$

$\boxed{x=7}$ or $x=-2$

$x=7$ is a solution

$x=-2$ is an extraneous root

d) $\left|\frac{x+2}{x+1}\right| \leq 2$

Case 1 $x+1>0$

$\frac{x+2}{x+1} \leq 2$ | $\frac{x+2}{x+1} \geq -2$

$x+2 \leq 2x+2$ | $x+2 \geq -2x-2$

$-x \leq 0$ | $3x \geq -4$

$\boxed{x \geq 0}$ | $x \geq -\frac{4}{3}$

Case 2 $x+1<0$

$\frac{x+2}{x+1} \leq 2$ | $\frac{x+2}{x+1} \geq -2$

$x+2 \geq 2x+2$ | $x+2 \leq -2x-2$

$-x \geq 0$ | $3x \leq -4$

$x \leq 0$ | $\boxed{x \leq -\frac{4}{3}}$

e) $|2x-7| \geq 15$

① $2x-7 \geq 15$

$2x \geq 22$

$\boxed{x \geq 11}$

② $2x-7 \leq -15$

$2x \leq -8$

$\boxed{x \leq -4}$

f) $12 > |x-5| > -8$

① $12 > x-5 > -8$

$17 > x > -3$

② $-12 < x-5 < 8$

$\boxed{-7 < x < 13}$

$\boxed{-3 < x < 17}$

③ a) $f(x) = \frac{x^2+5x}{x^2+10x+25} = \frac{x(x+5)}{(x+5)(x+5)} = \frac{x}{x+5}$

① roots: $x=0$

② V.A $x=-5$

③ H.A. $y=1$

④ Holes: None

⑤ y int: $y=0$